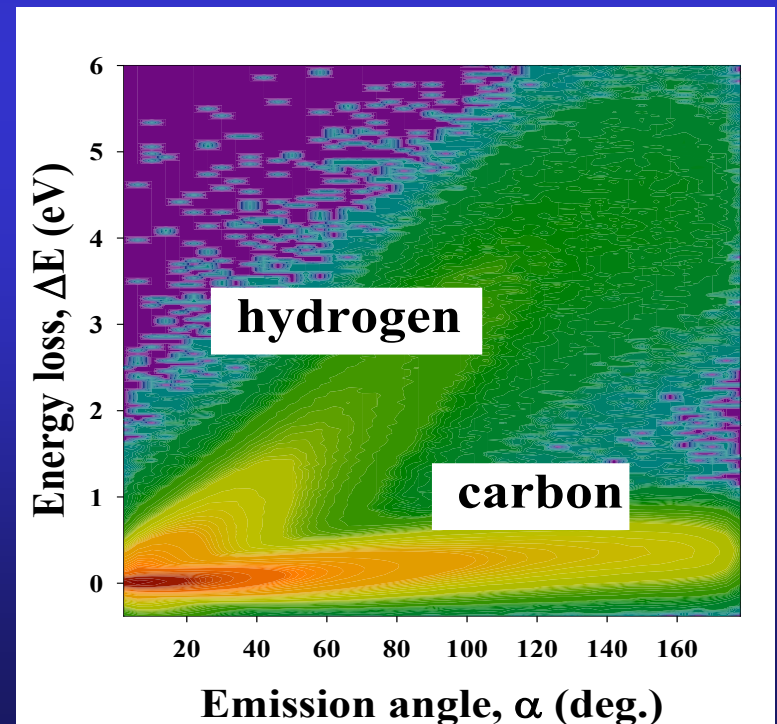
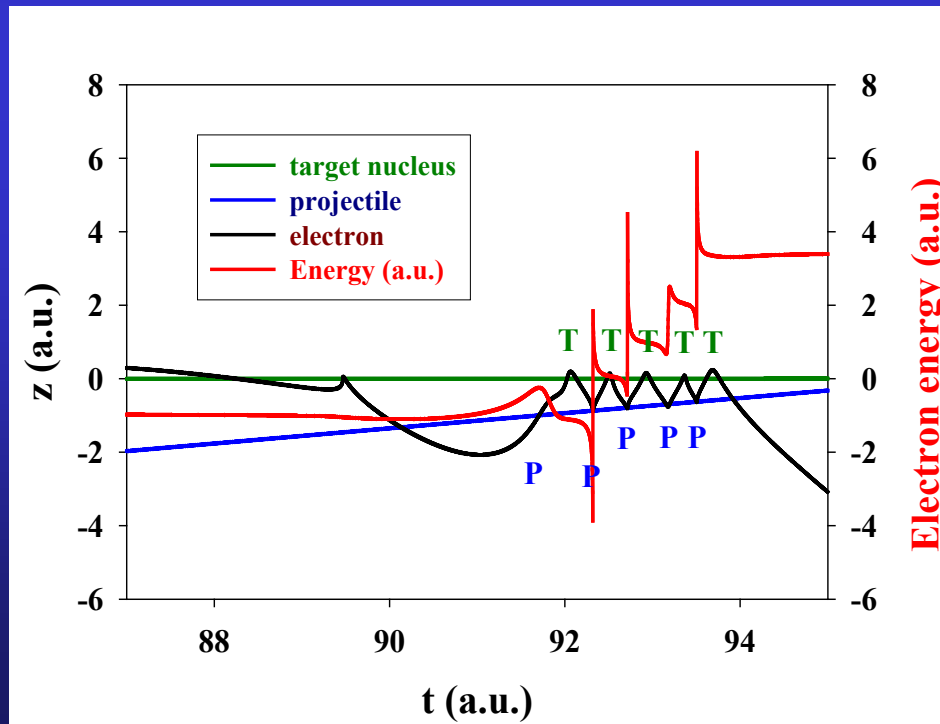


# Multiple electron scattering in ion-atom and electron-solid collisions

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# Outlook

## Fermi-shuttle type multiple electron scattering in atomic collisions

Basic idea --- Why? ---History

Classical Trajectory Monte Carlo method -- a method of the analysis

The present level of understanding -- Examples

Summary

## Monte Carlo simulation of electron spectra backscattered elastically from solid sample

Why? --- Background and motivation

Monte Carlo simulation

Results

- Energy loss and angular correlation pattern
- Energy loss distributions
- Single and multiple scattering

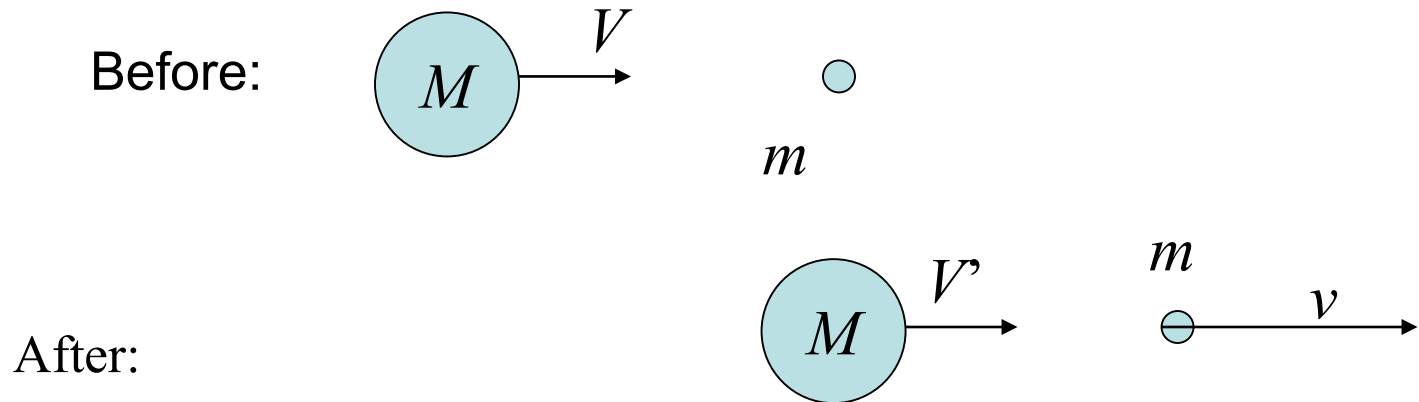
Summary

## **Part I:**

# **Fermi-shuttle type multiple electron scattering in atomic collisions**

# Ping-pong game: heavy paddle – light ball

## Elastic scattering:



**Momentum conservation:**  $MV = MV' + mv$

**Energy conservation:**  $\frac{1}{2}MV^2 = \frac{1}{2}MV'^2 + \frac{1}{2}mv^2$

$$V' = V \frac{1 - m/M}{1 + m/M}$$

$$v = 2V \frac{1}{1 + m/M}$$

The final velocity of the light particle in the laboratory frame

Large energy gain

# Energy gain in ping-pong game

**Projectile velocity (V)**

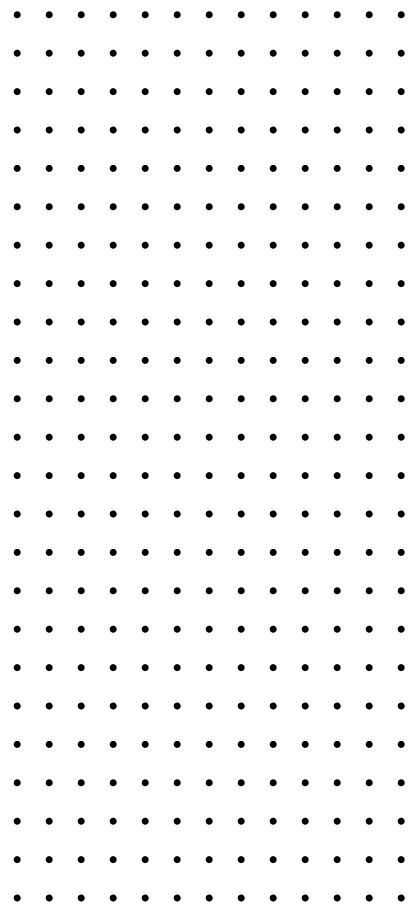
$$E_V = 0.5 m_e V^2$$

<b>kicks:</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>ball velocity:</b>	<b>2V</b>	<b>4V</b>	<b>6V</b>	<b>8V</b>	<b>10V</b>
<b>ball energy:</b>	<b>4 E<sub>V</sub></b>	<b>16 E<sub>V</sub></b>	<b>36 E<sub>V</sub></b>	<b>64 E<sub>V</sub></b>	<b>100 E<sub>V</sub></b>

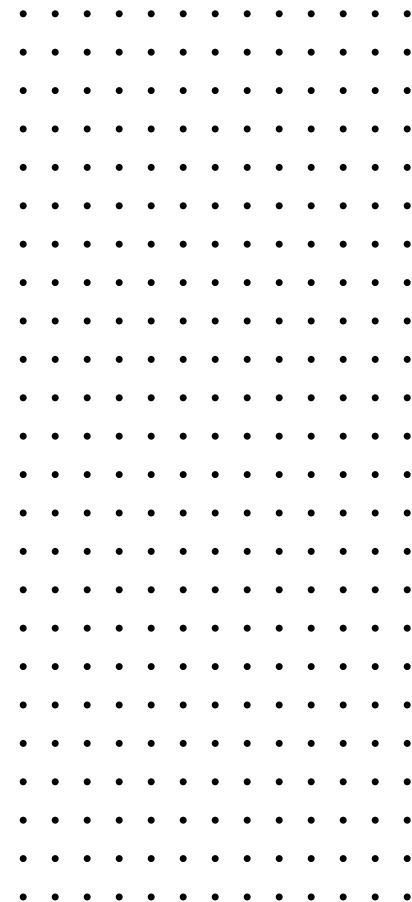
# Why?

- Basic research - fundamental physics
- Hot electrons
  - astrophysics
  - ion-beam technology
- Technological importance
  - plasma physics
  - fusion
  - analytical methods – medical application**

# Charge particles in moving magnetic fields



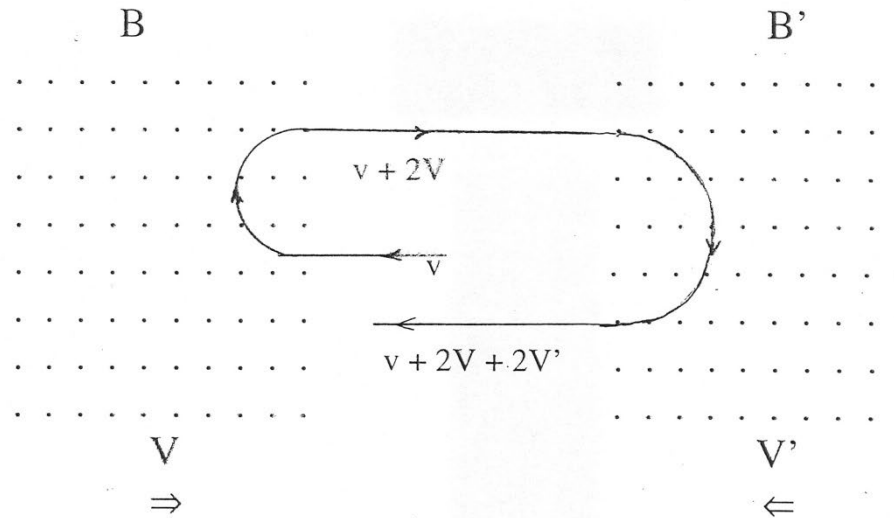
$B_1$



$B_2$

## Pioneer: E. Fermi, Phys Rev. 75 (1949)

A possible origin of cosmic rays (energetic charged particles):



Typical values:

B:  $\sim 10^{-5}$  gauss

V:  $\sim 30$  km/s

E-gain:  $\sim 10$  eV / reflection (in the nonrelativistic regime)

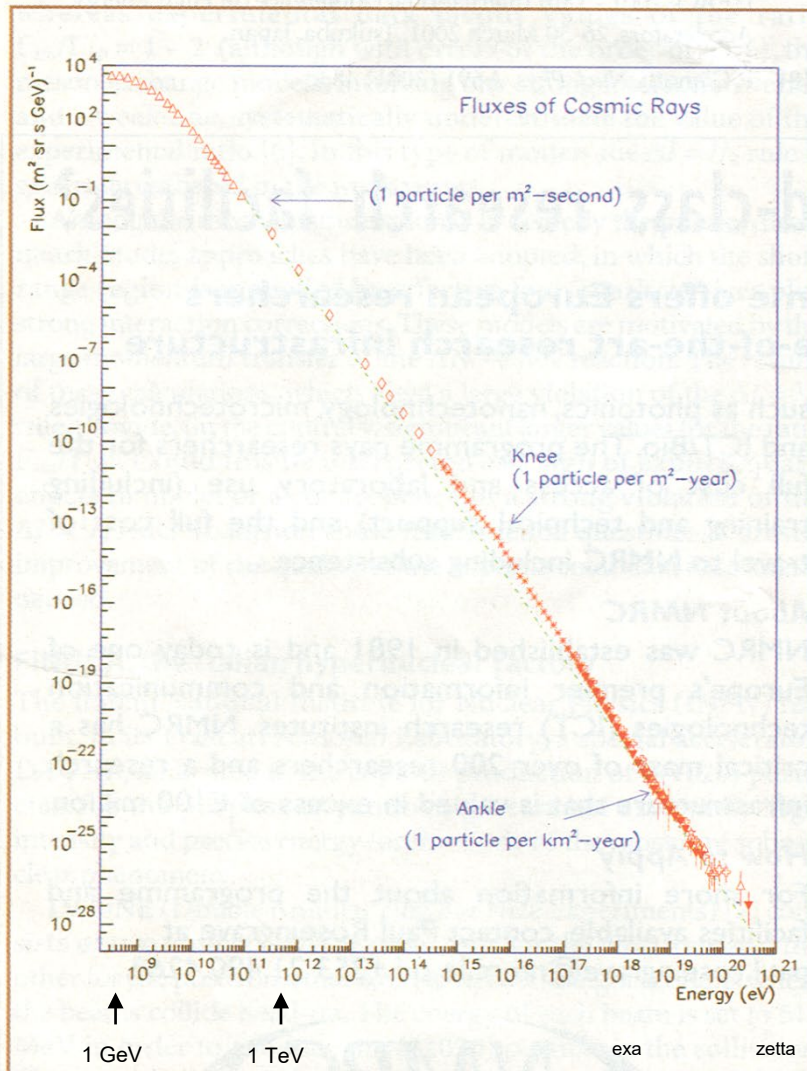
Number of collisions:  $\sim 10^8$

Minimum injection energy:  $\sim 200$  MeV (for protons)

Final energy: 1-2 GeV

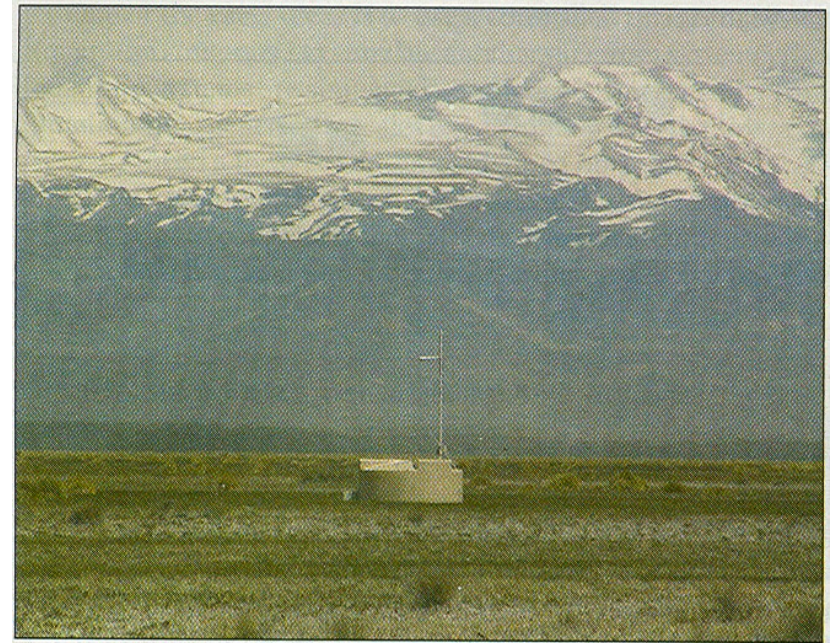


# Energy distribution of the cosmic particles (particle / (m<sup>2</sup> sr s GeV))



▲ Fig. 1: The all-particle spectrum of cosmic rays (from S. Swordy). The arrows and values between parentheses indicate the integrated flux above the corresponding energies.

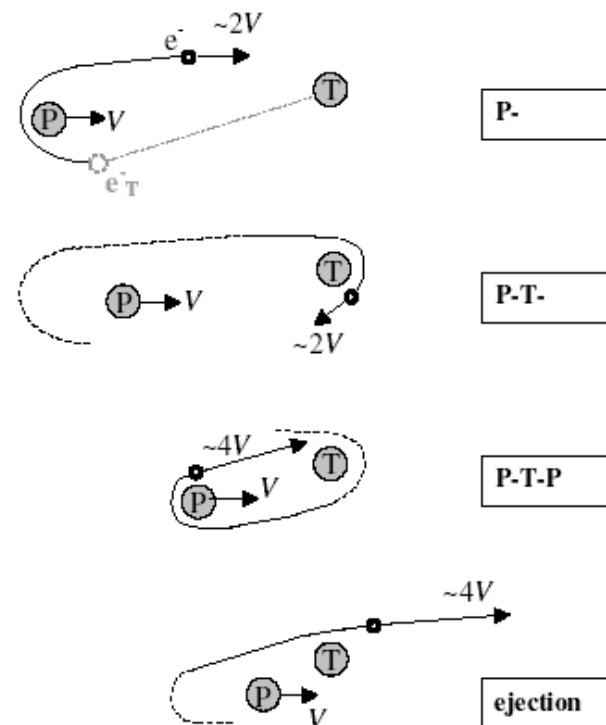
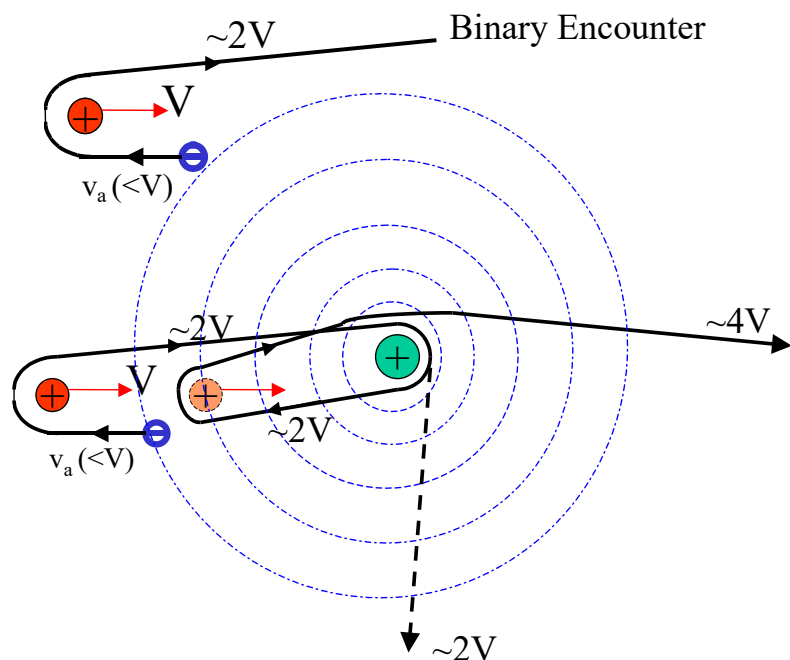
**Pierre Auger project - Argentina**  
**1600 detectors in 3000 km<sup>2</sup>**



Source: M. Boratav, Probing theories with Cosmic rays  
Europhysics News, September/October (2002), 162

# Mechanism

# Movie



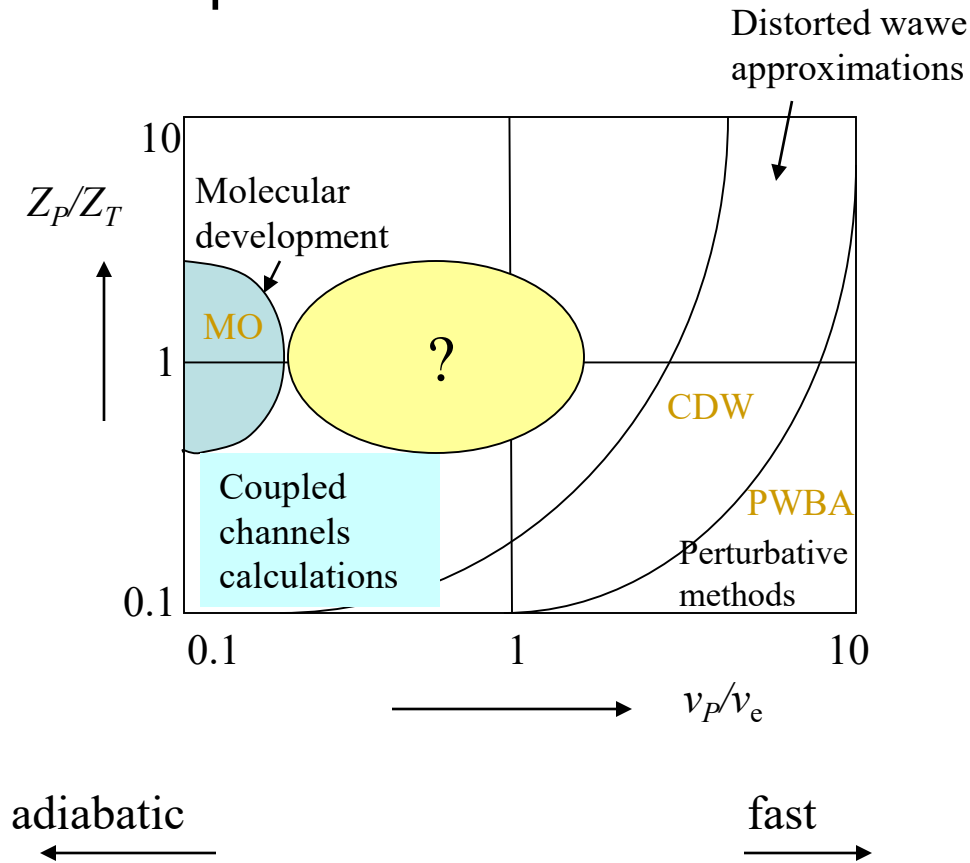
Start with:            target ionization:            projectile ionization (loss):  
 $v_e = 2V, 4V, 6V, \dots$     or     $V, 3V, 5V, \dots$

## References:

1. B. Sulik *et al.*, Phys. Rev. Lett. **88**, 73201(2001),
2. B. Sulik, K. Tőkési, Advances in Quantum Chemistry **52** (2007) 253.

# Ionization in ion-atom collisions

## Description:

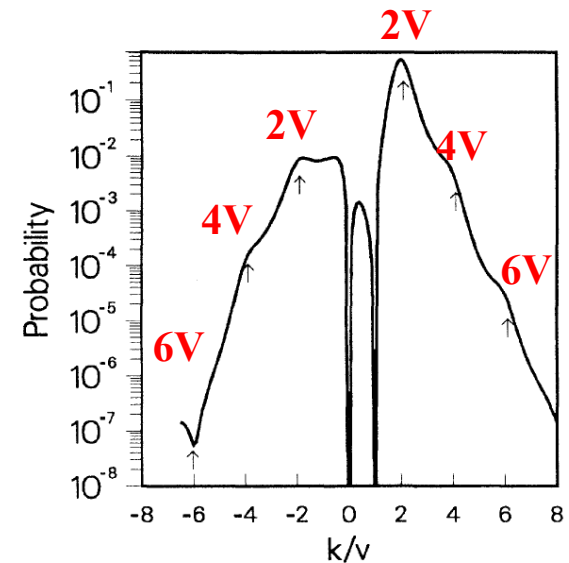


## Non-perturbative models:

Classical (CTMC)

Exact quantum models, e.g., one dimensional „scattering” on a delta potential

Surprise (Wang et al., 1991):

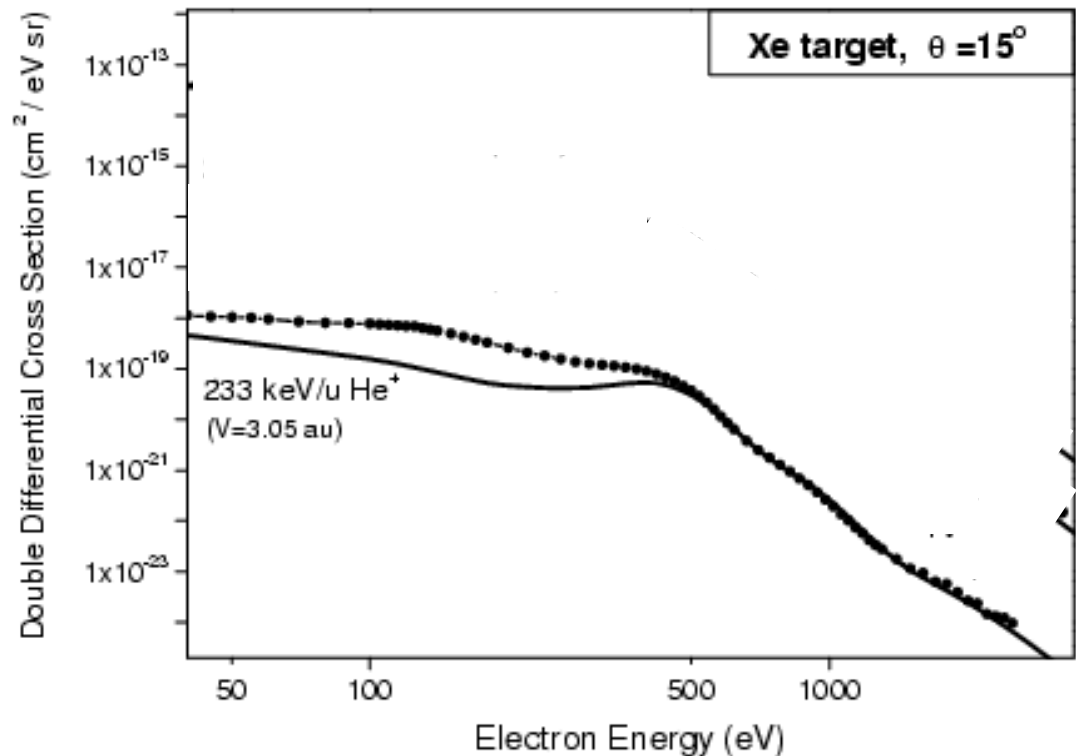


# Observation of the Fermi-shuttle process in the double-differential electron spectra. **Separation of multiple scattering components.**

1st order Born theory is excellent for light ion impact at high electron energies.

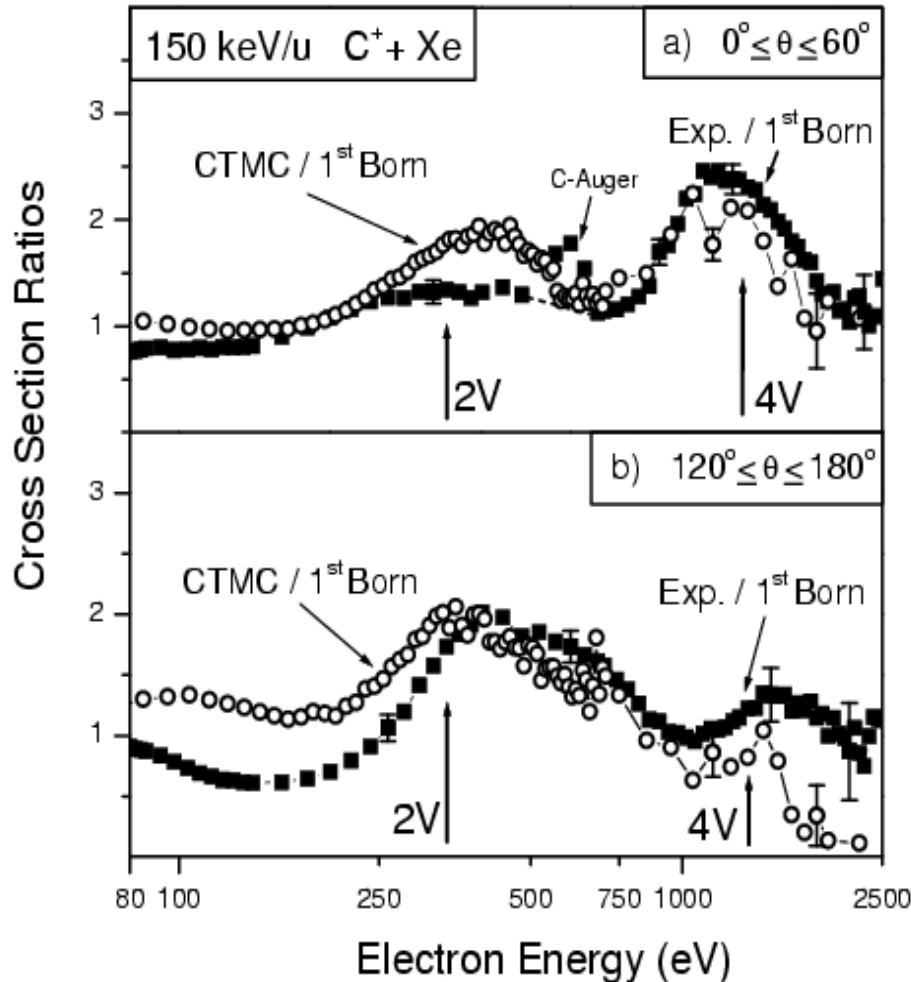
Higher-order process is on the background of 1st order processes

For carbon ions we have extra yields above the first order calculations, centered at **4 V**

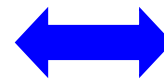


**Key for identification: kinematics**

# Integrated cross sections in forward and backward angles



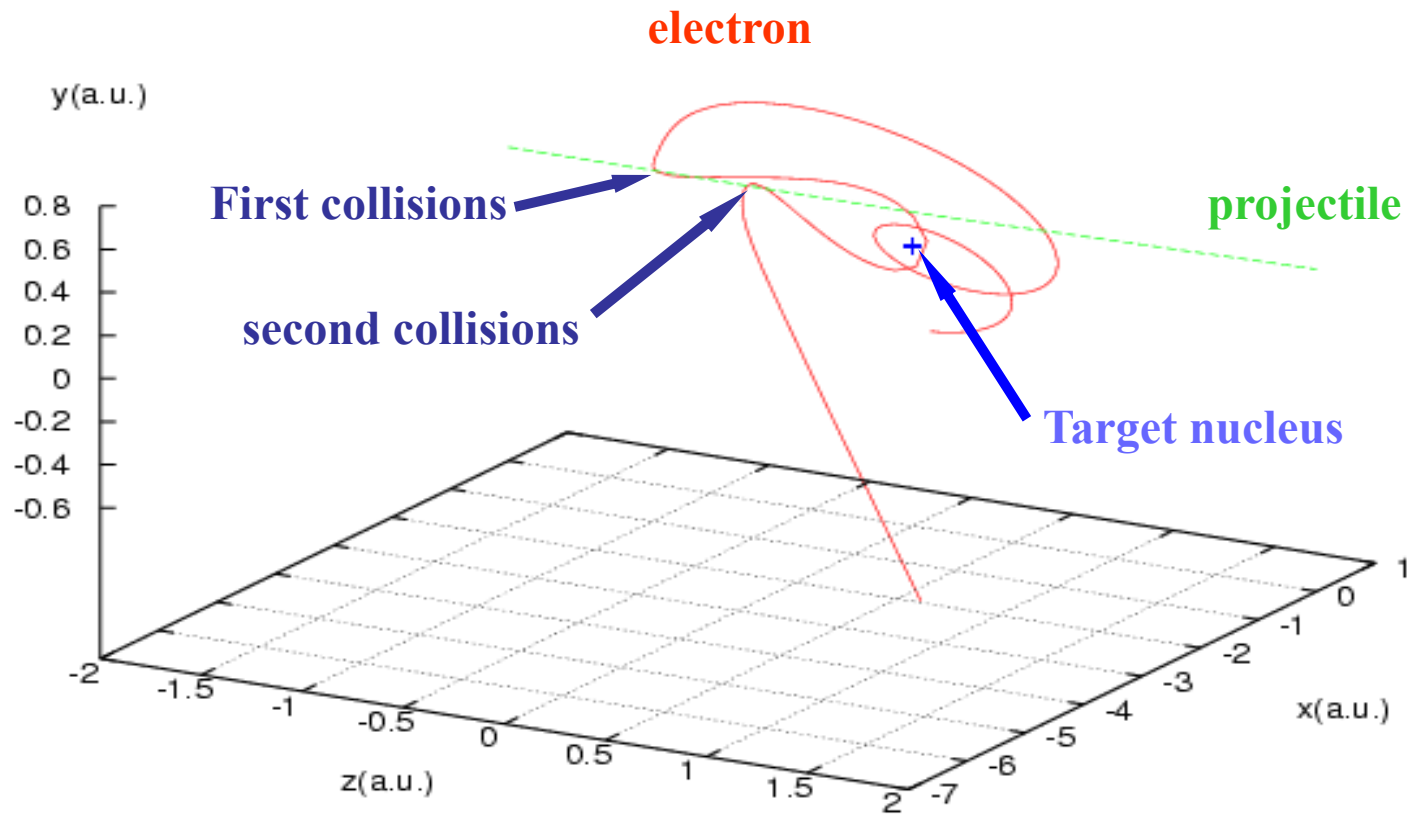
Exp/Born



CTMC/Born

for Xe 3d, 4d, 5p shells

# Example

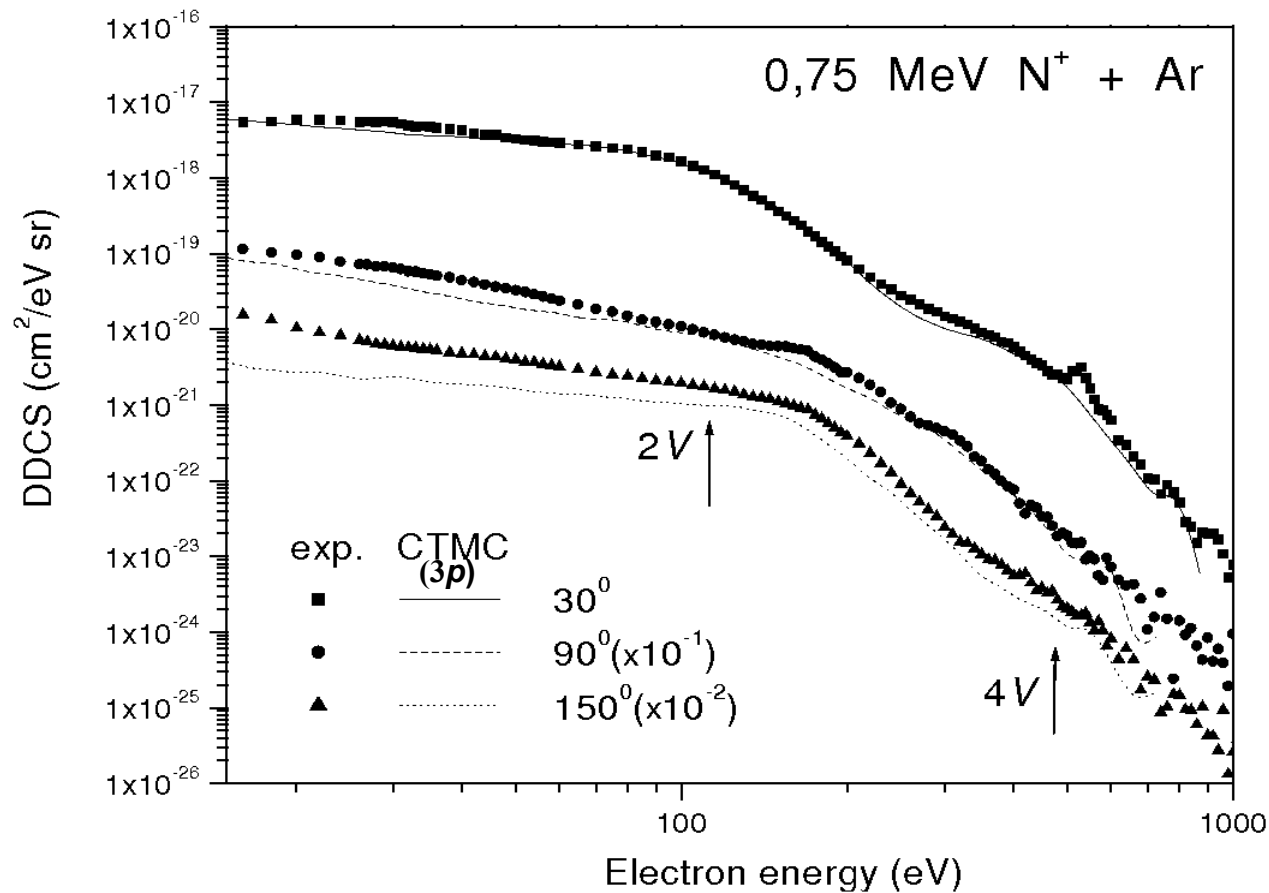


1.8 MeV (150 keV/u) C<sup>+</sup> + Xe  
 Statistics for ping-pong  
 Energy window: 1100-1500 eV

Sub-shell	0-30 degree <i>forward emission</i>		150-180 degree <i>backward emission</i>	
	<u>events</u>	<u>P-T-P</u>	<u>events</u>	<u>P-T-P-T</u>
3d	45	84%	29	90%
4d	80	80%	26	65%
5p	21	71%	4	75%

# Somewhat higher ion impact energies – 2

## Absolute cross sections

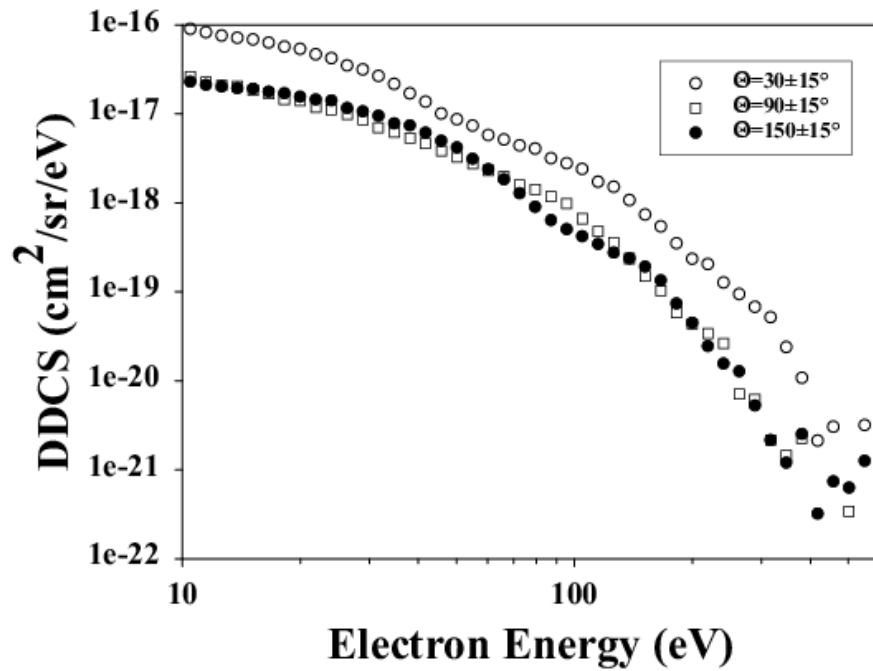


→ 85% P-T-P and P-T-P-T

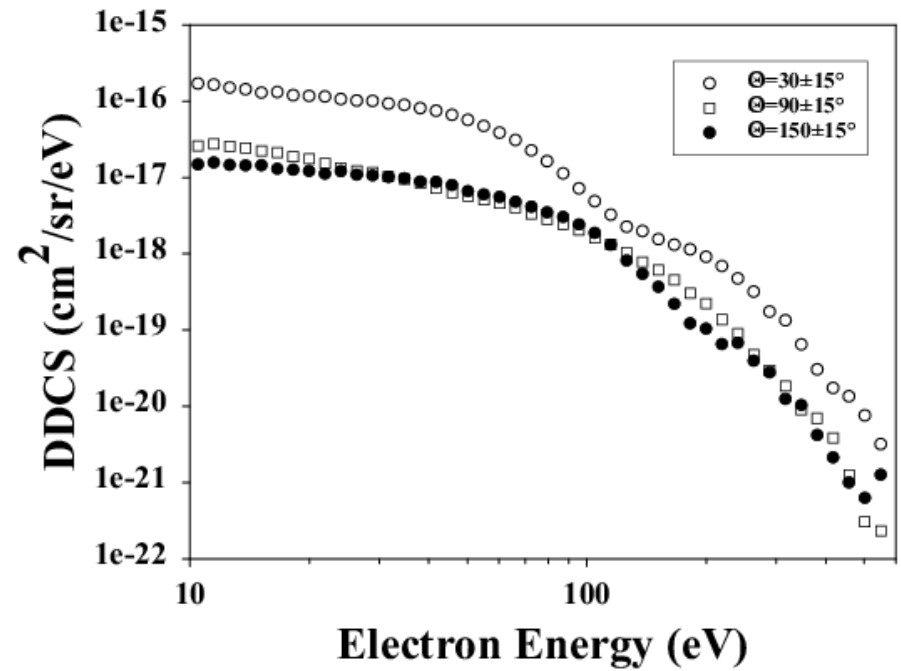


# CTMC results

200 keV  $O^+$  + Ar(3p)  
electron emission  
calculated by CTMC

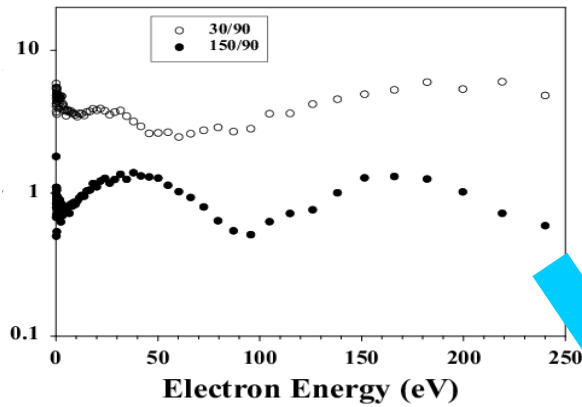


400 keV  $O^+$  + Ar(3p)  
electron emission  
calculated by CTMC

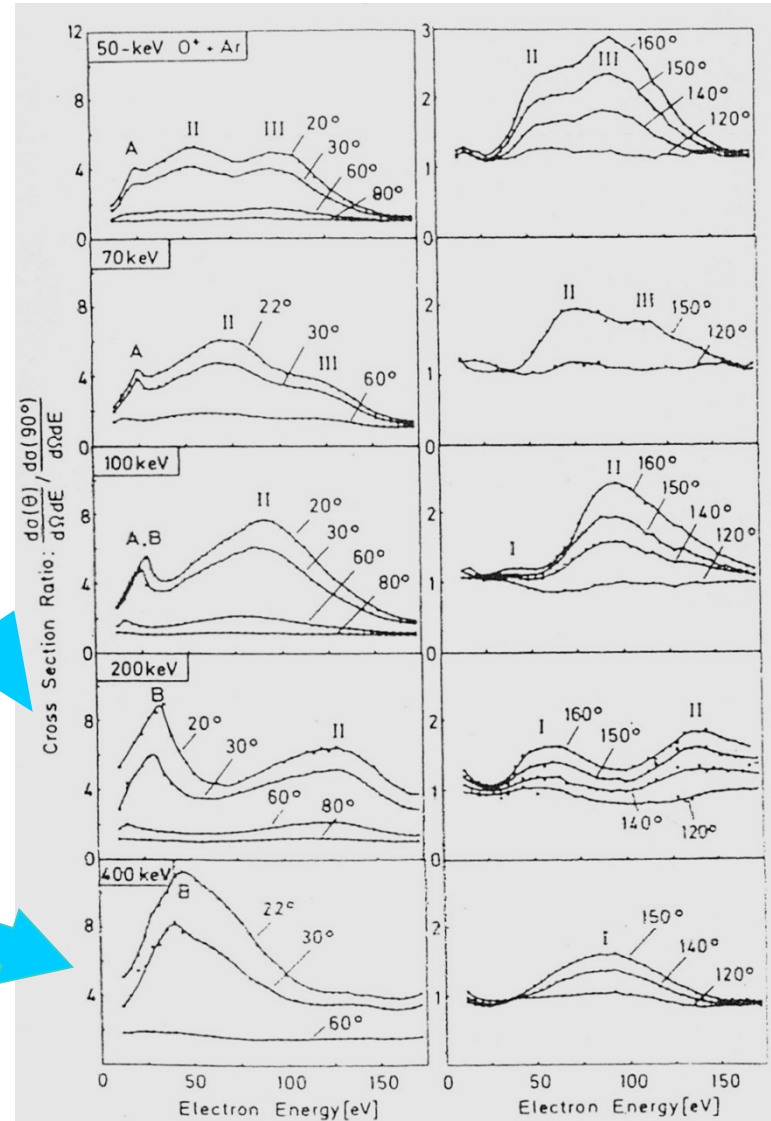
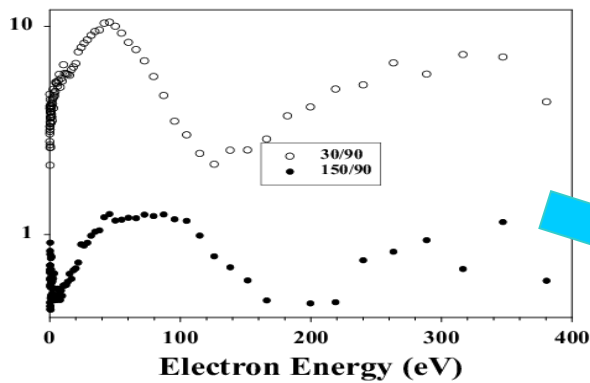


# CTMC results - ratios

200 keV  $O^+ + Ar(3p)$   
electron emission  
calculated by CTMC

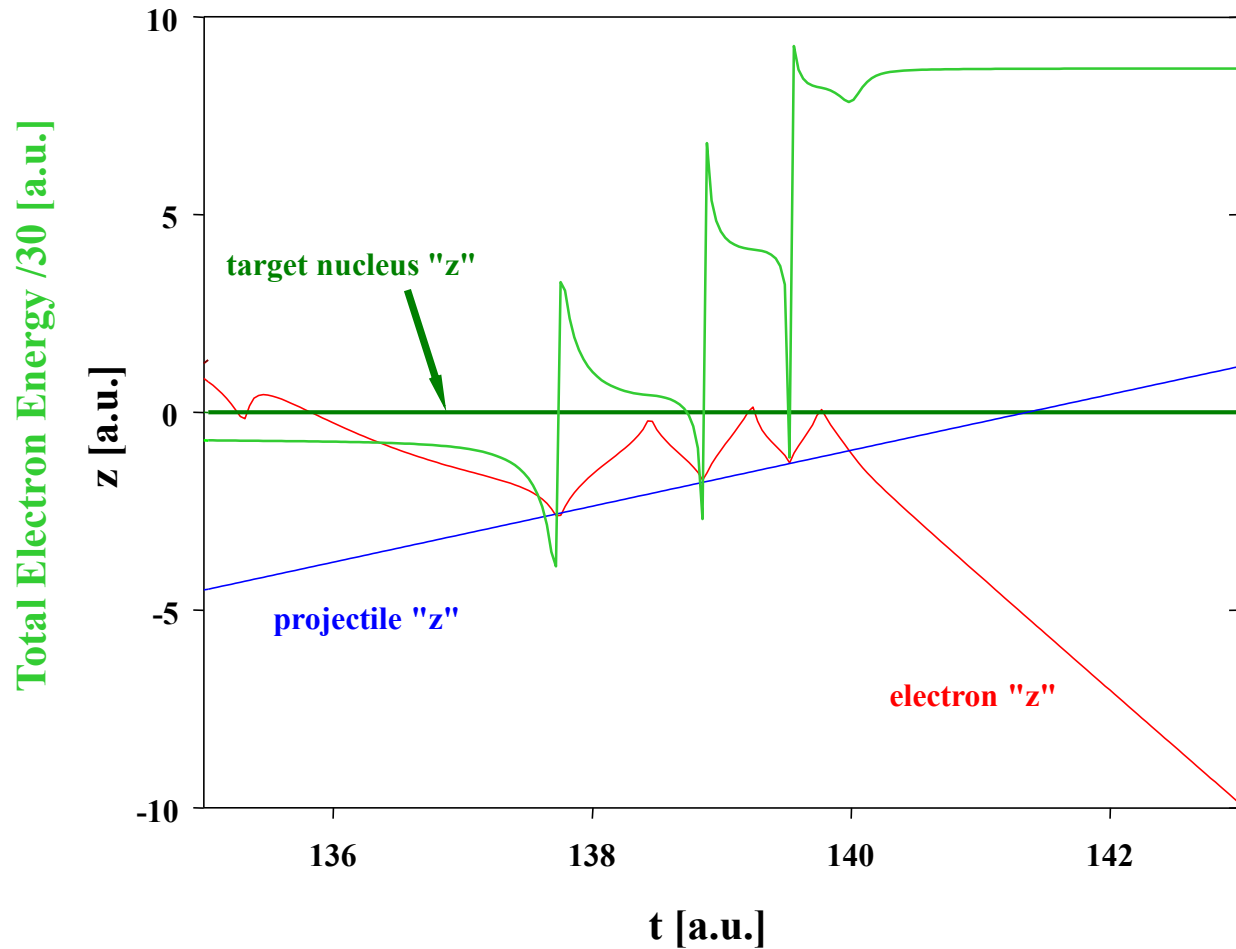


400 keV  $O^+ + Ar(3p)$   
electron emission  
calculated by CTMC



# CTMC trajectories

200 keV  $O^+$  + Ar(3p)  
 $E_{\text{electron}} = 260 \text{ eV}$ ,  $\theta = 155^\circ$   
 $b = 0.91 \text{ a.u.}$

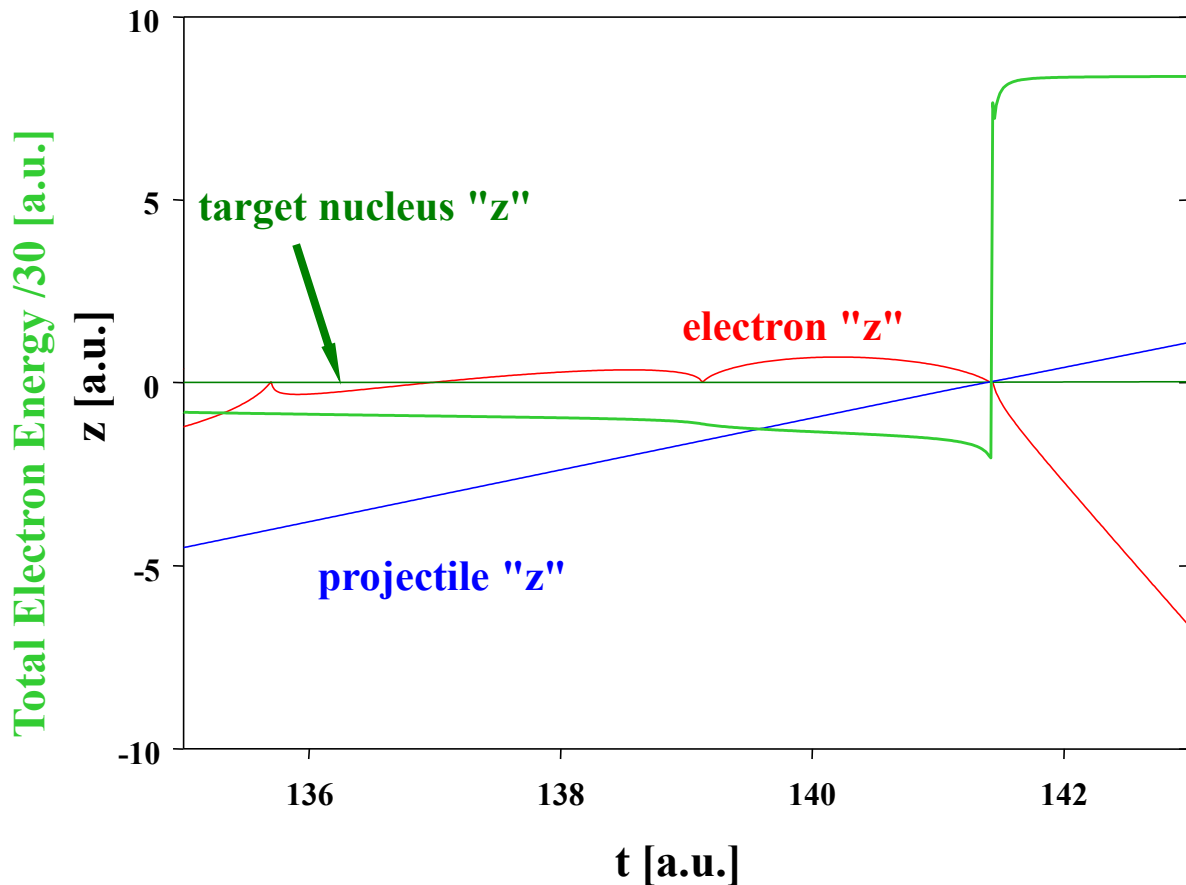


# CTMC trajectories

200 keV  $O^+$  + Ar(3p)

$E_{\text{electron}}=250$  eV,  $\theta=155^\circ$

$b=0.07$  a.u.



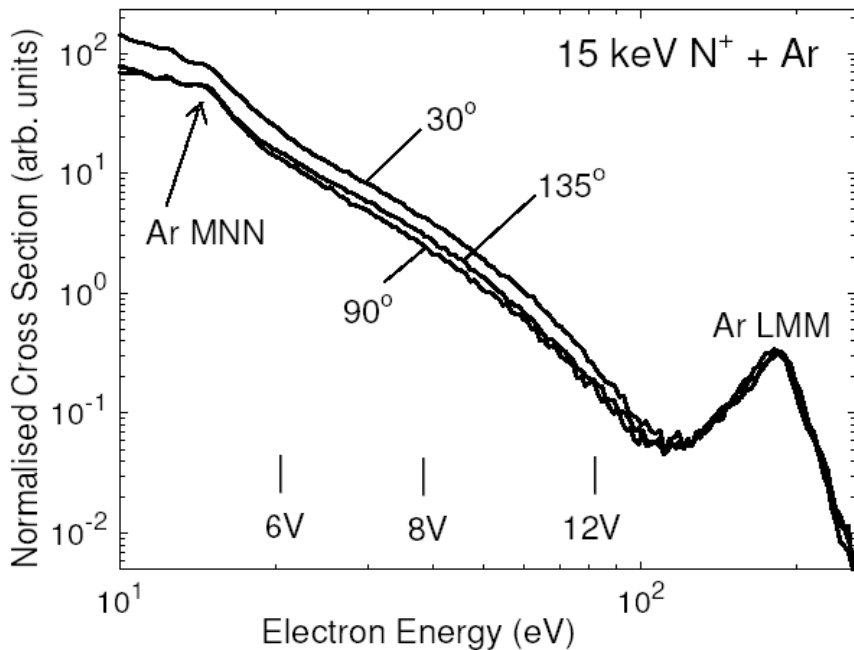
Estimated ratio:  $< 10\%$   
(by sampling)  
(for  $E_e > 100$  eV)

The majority of events looks to be accelerating scattering (or – with other words – non-adiabatic quasimolecular development)

# Slow ion impact (>98% ping-pong)

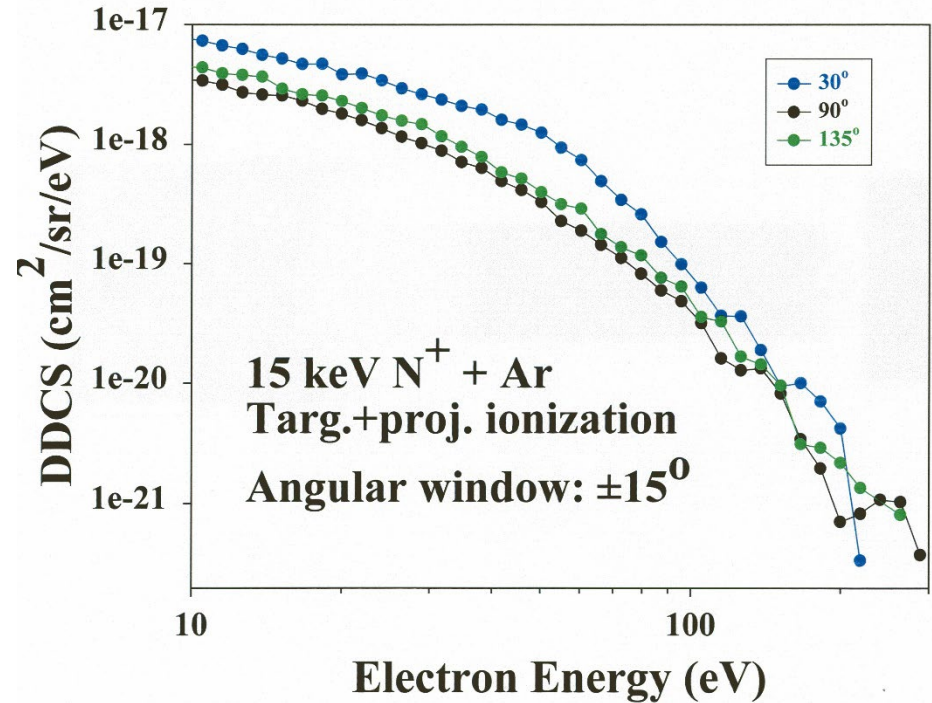
**Experiment**

**HMI Berlin**



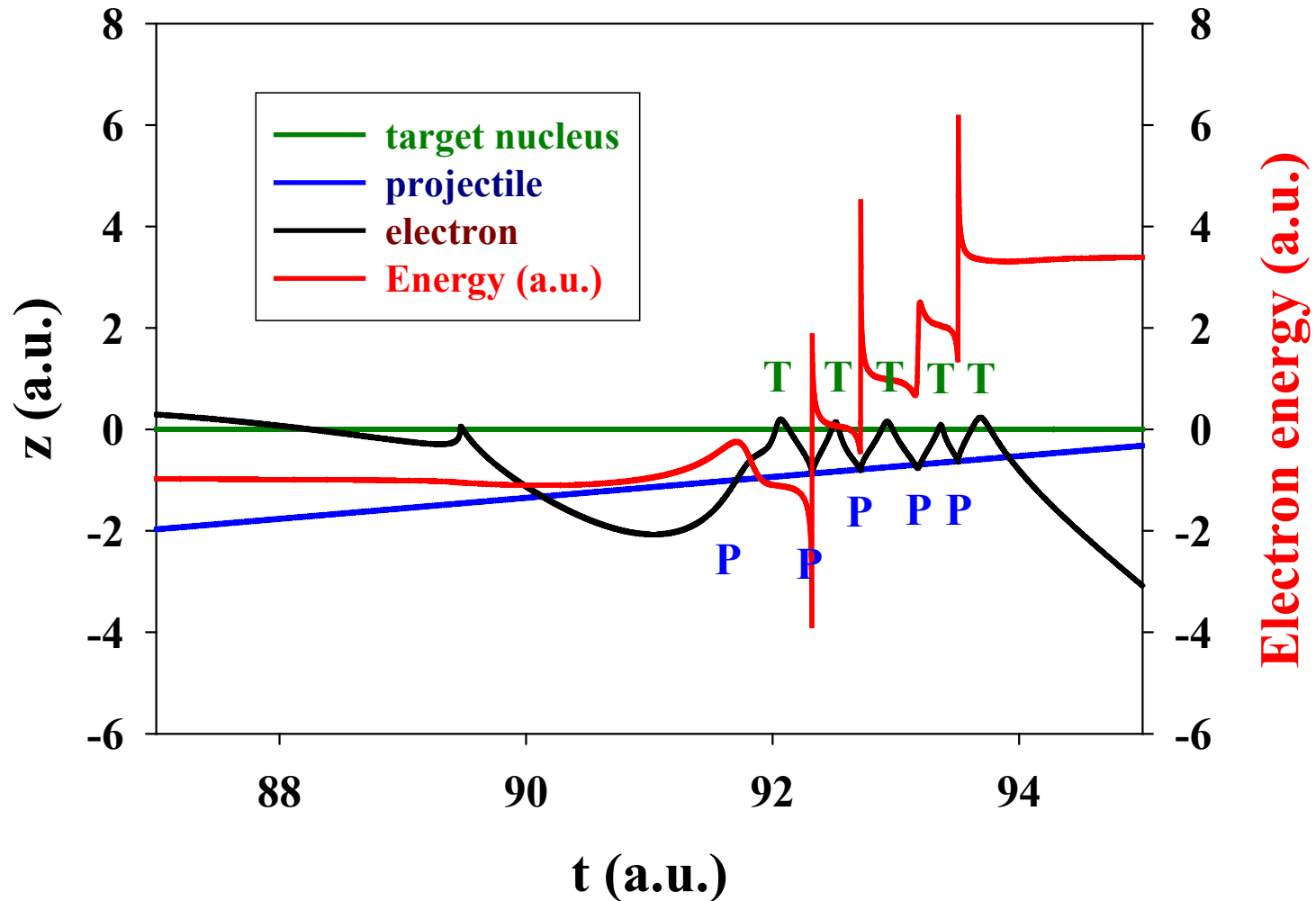
**CTMC**

**Debrecen**



# Long ping-pong game (15 keV N<sup>+</sup> + Ar)

P-T-P-T-P-T-P-T-P-T



# Conclusions

- CTMC reproduce different experiments for collisions of slow, singly charged ions (1-30 keV/u) with atoms.
- A method for analysing the CTMC „events” has been developed.

**-Fermi-shuttle multiple scattering is significant or dominant for slow collisions.**

**Electron emission in low energy ion-matter interactions might be governed by multiple scattering.**

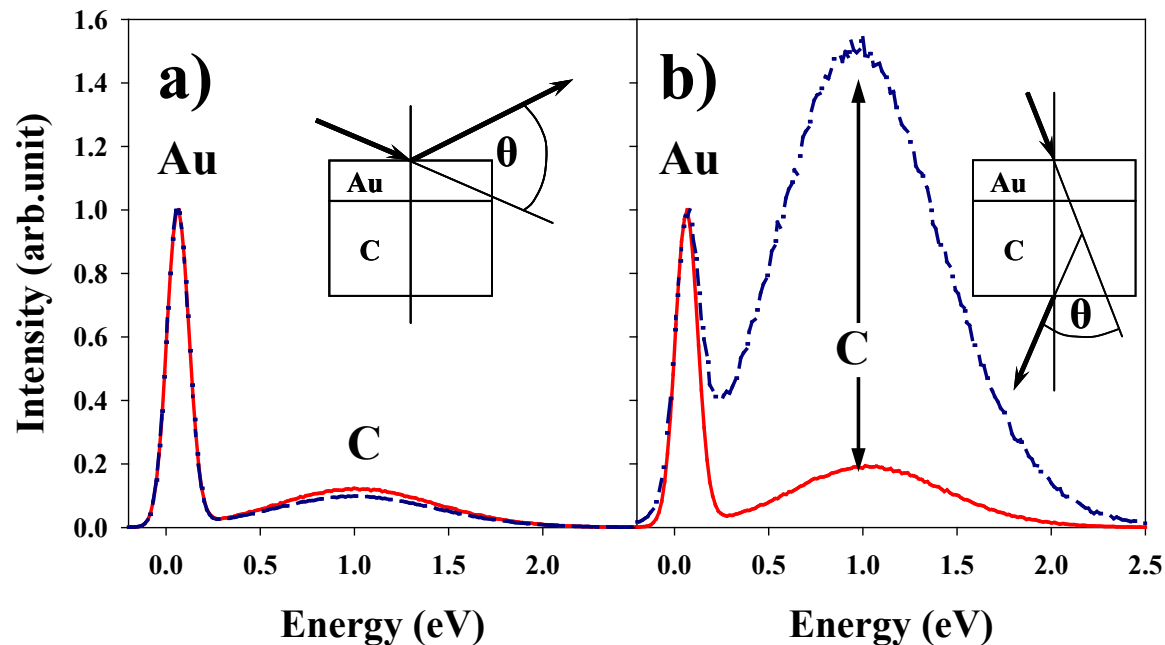
## Part II:

Monte Carlo simulation of electron spectra  
backscattered elastically from solid sample

**The influence of multiple scattering**

**Double layers**

**Multicomponent samples**





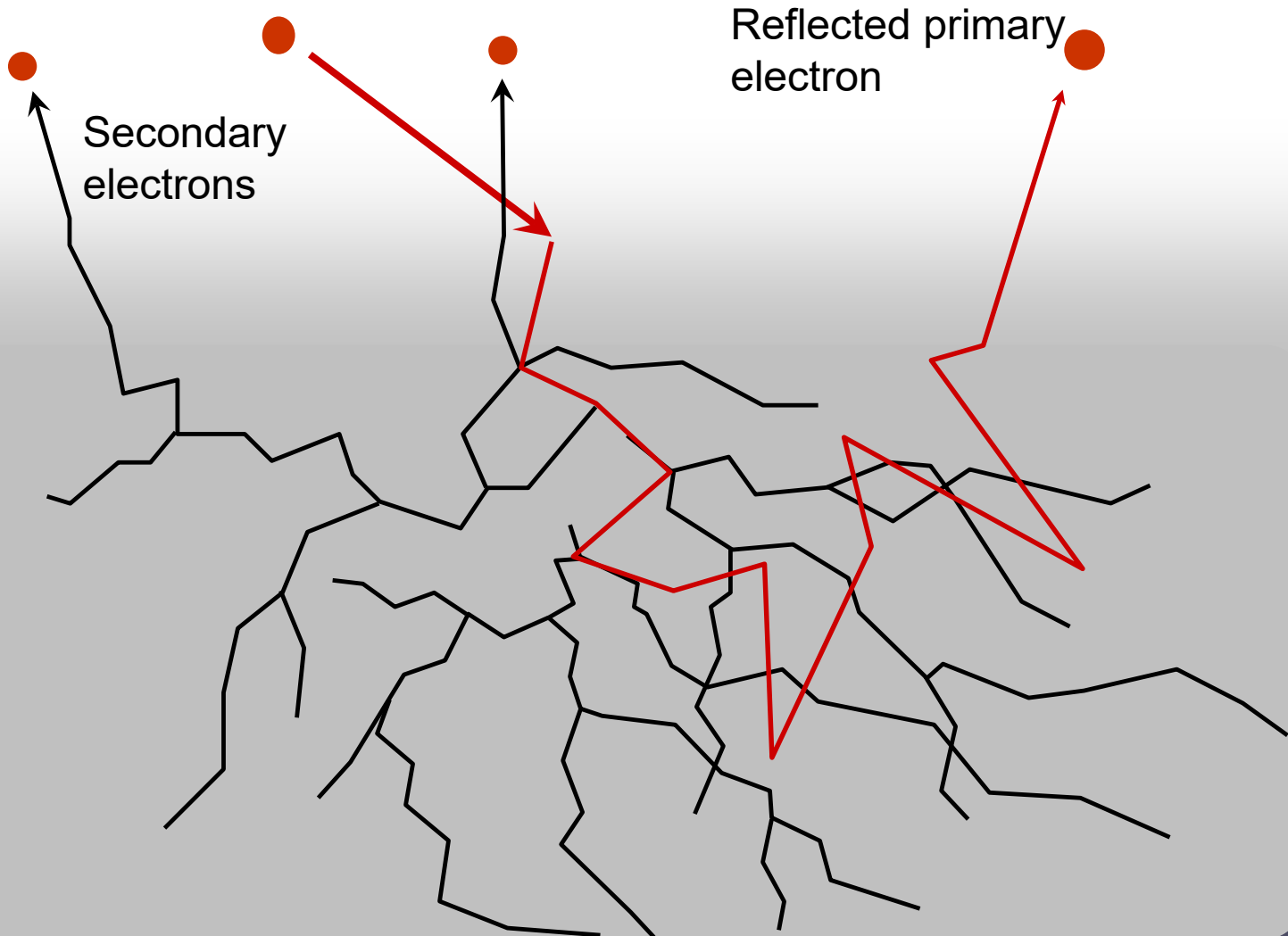
# Why?

We take an advantage that the energy of the elastically backscattered electrons is shifted from the primary values due to the energy transfer between the primary electron and the target atoms (recoil effect).

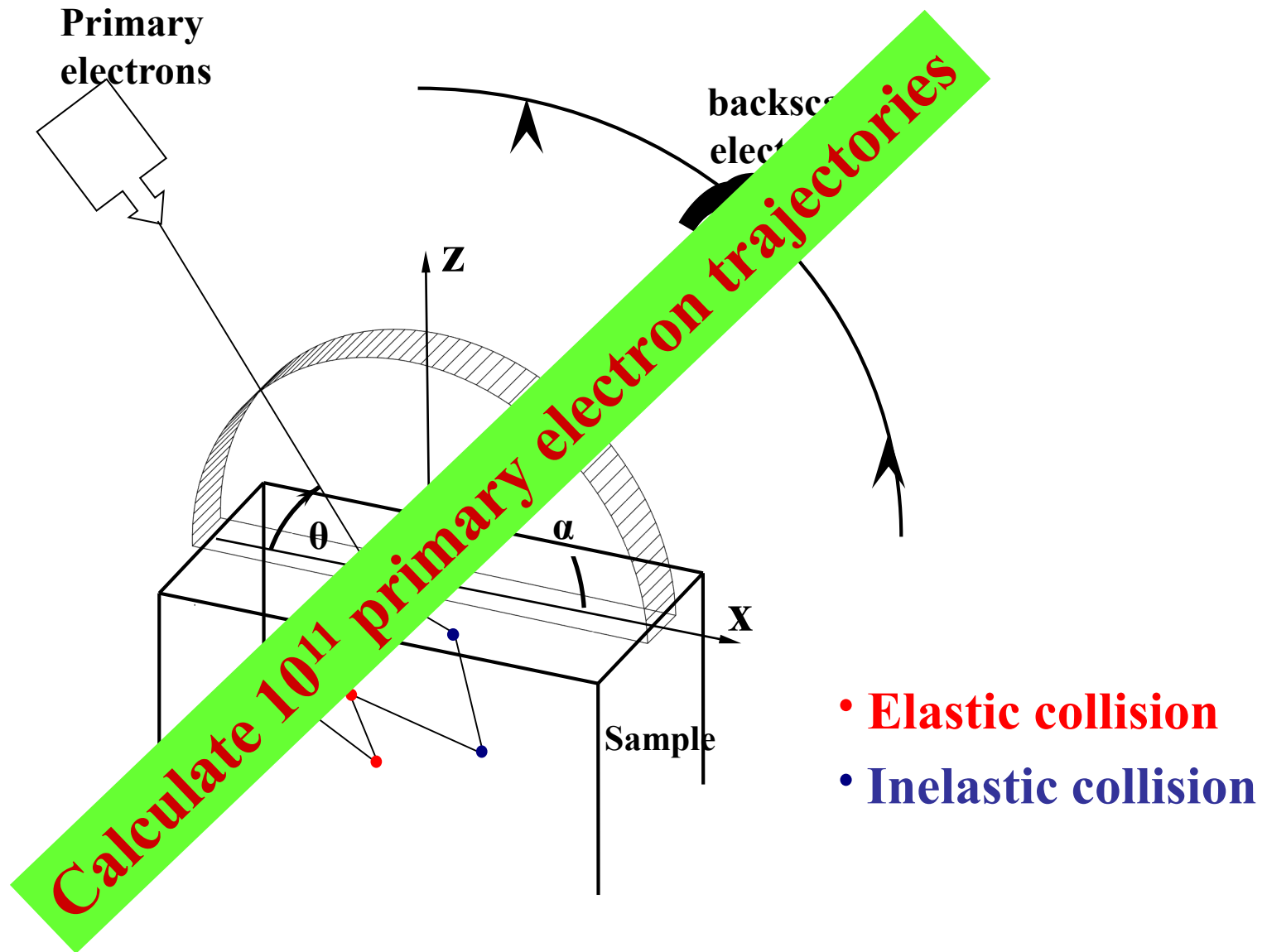
## Application for quantitative analysis

- hydrogen cell → new energy source
- measuring in nano-scale

# Scenario



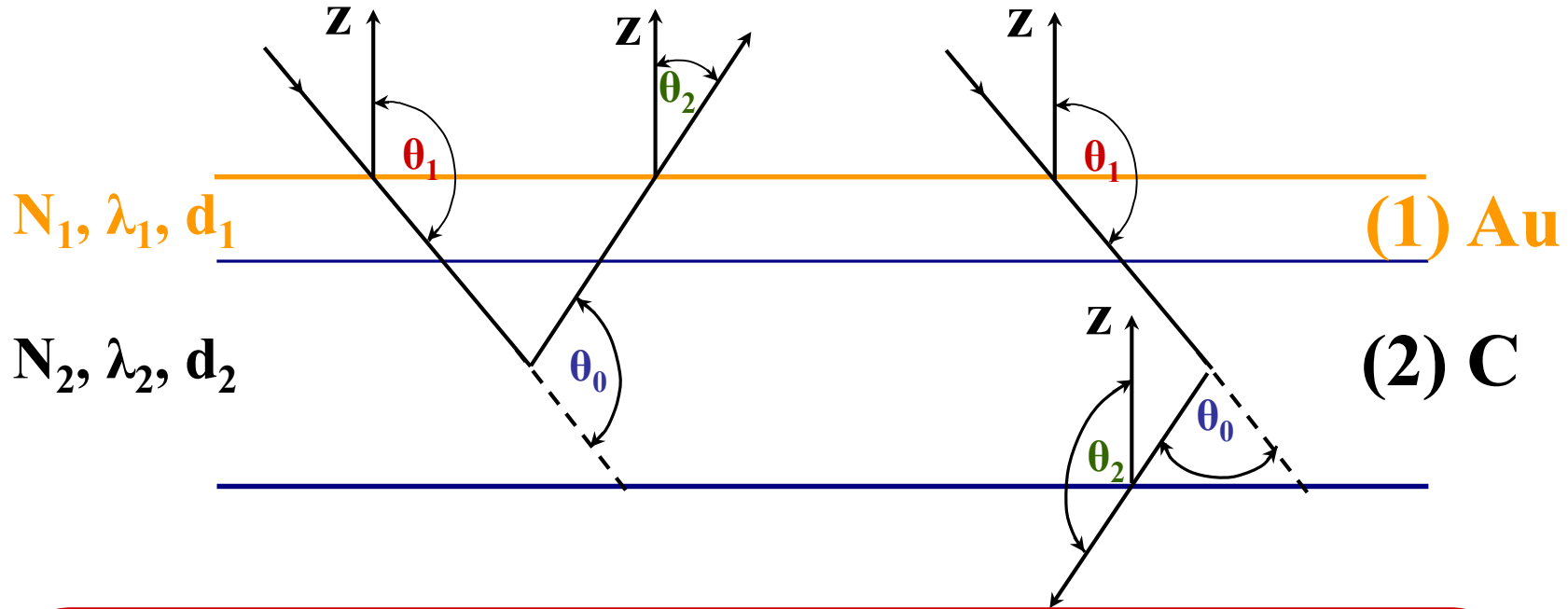
# Schematic view of the geometric configuration of the calculation.



# Scattering Geometry

**Backscattering**

**Transmission**



$$dP_1(1) \left\{ \begin{array}{l} dP_1(1) = g e^{d_1/(\lambda_1 \cos \theta_2)} e^{d_2/(\lambda_2 \cos \theta_2)} [1 - e^{d_1/(\lambda_1 g \cos \theta_1)}] \lambda_1 N_1 \frac{d\sigma_1}{d\Omega}(\theta_0) d\Omega, \\ dP_1(2) \left\{ \begin{array}{l} dP_1(2) = g e^{d_1/(\lambda_1 \cos \theta_1)} e^{d_2/(\lambda_2 \cos \theta_2)} [1 - e^{d_2/(\lambda_2 g \cos \theta_1)}] \lambda_2 N_2 \frac{d\sigma_2}{d\Omega}(\theta_0) d\Omega, \end{array} \right. \end{array} \right.$$

**where**

$$g = \frac{\cos \theta_2}{\cos \theta_2 - \cos \theta_1}$$

# Monte Carlo simulation

- **Monte Carlo simulation of electron transport in solids is based on the stochastic description of scattering processes.**
- **Electron penetration is approximated by a classical zigzag trajectory.**
- **In our simulations both the elastic and inelastic scattering events were taken into account.**
- **For the case of the first inelastic collision the calculations were stopped.**
- **Particular values of scattering angles of electrons in an individual event are realized by random numbers following the angular differential elastic cross sections of carbon and gold.**

# Energy loss due to elastic scattering

$$E_r = \frac{2mE_0}{M_l} \left( 1 + \frac{E_0}{2mc^2} \right) \left[ 1 - \cos\theta_0 + \sqrt{\frac{M_l \varepsilon_l}{m \varepsilon_0} \left( 1 - \frac{E_0}{2mc^2} \right)} (\cos\theta_l - \cos\theta_0 \cos\theta_l - \sin\theta_0 \sin\theta_l \cos\varphi_l) \right]$$

- $m$  and  $E_0$  are the mass and kinetic energy of the electron
- $\theta_0$  is the scattering angle
- $l = 1, 2$  denotes the kind of atom taking part in the collision
- $M_l, \varepsilon_l$  are the mass and kinetic energy of the atom
- $\theta_l$  és  $\varphi_l$  characterizes the direction of motion of the atom with respect to the velocity of the electron before the scattering event

**Monte Carlo simulation of electron  
spectra backscattered elastically from  
double-layer sample at relativistic  
energy**

## Background

- **Studies of Electron Rutherford Backscattering (ERBS) are in the centre of interest.**

**elastic scattering of electrons (3-40 keV)**

**- good energy resolution ( $< 1\text{eV}$ )**

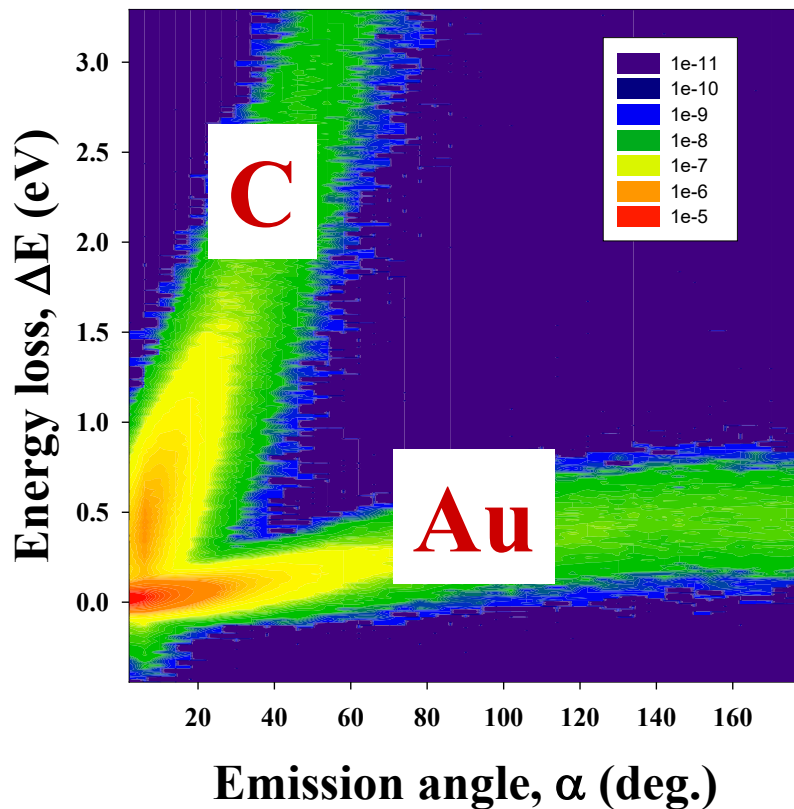
## Motivation

- **Accurate Monte Carlo simulation for double-layer system.**
- **Effects of the multiple and „mixed” scatterings to the elastic peak.**
- **peak intensities - estimate of the thickness layer**
- **FWHM of the peak – average kinetic energy of the electrons in solid**
- **accurate peak shape – final state interaction**

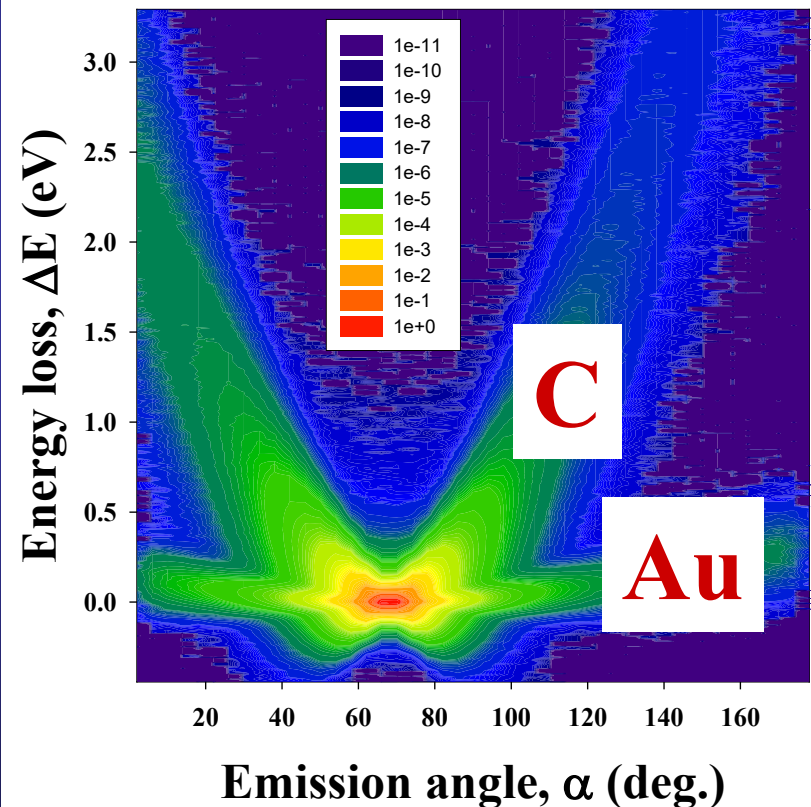


Contour plot (blue: minimum intensity, red: maximum intensity) of the electron intensity of elastically scattered electrons from Au-C double-layer  
Au - 1Å, C- 90 Å.

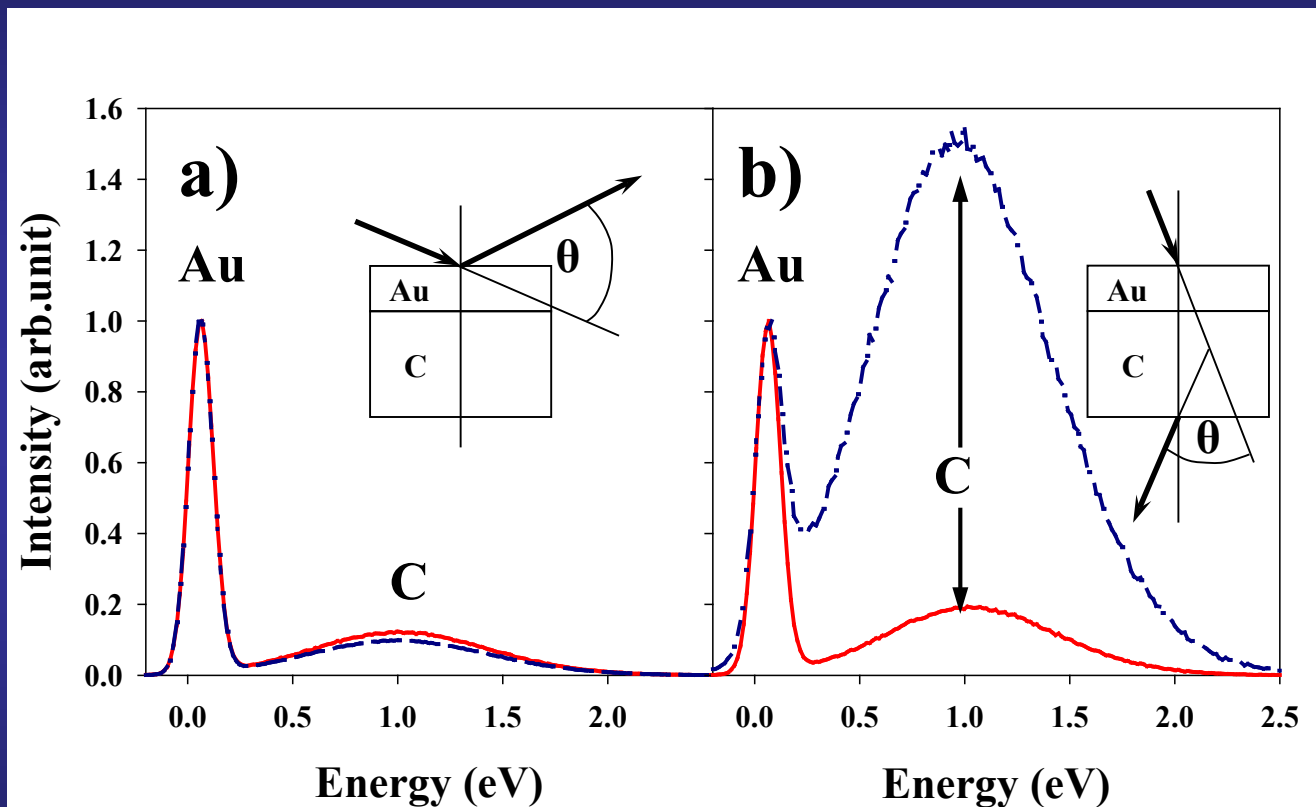
backscattered 40 keV mirror type



Transmitted 40 keV mirror type



# Energy loss distributions at 40 keV primary energy $\theta_0 = 44.3^\circ$ and $\Delta\Omega = \pm 5^\circ$ solid angle

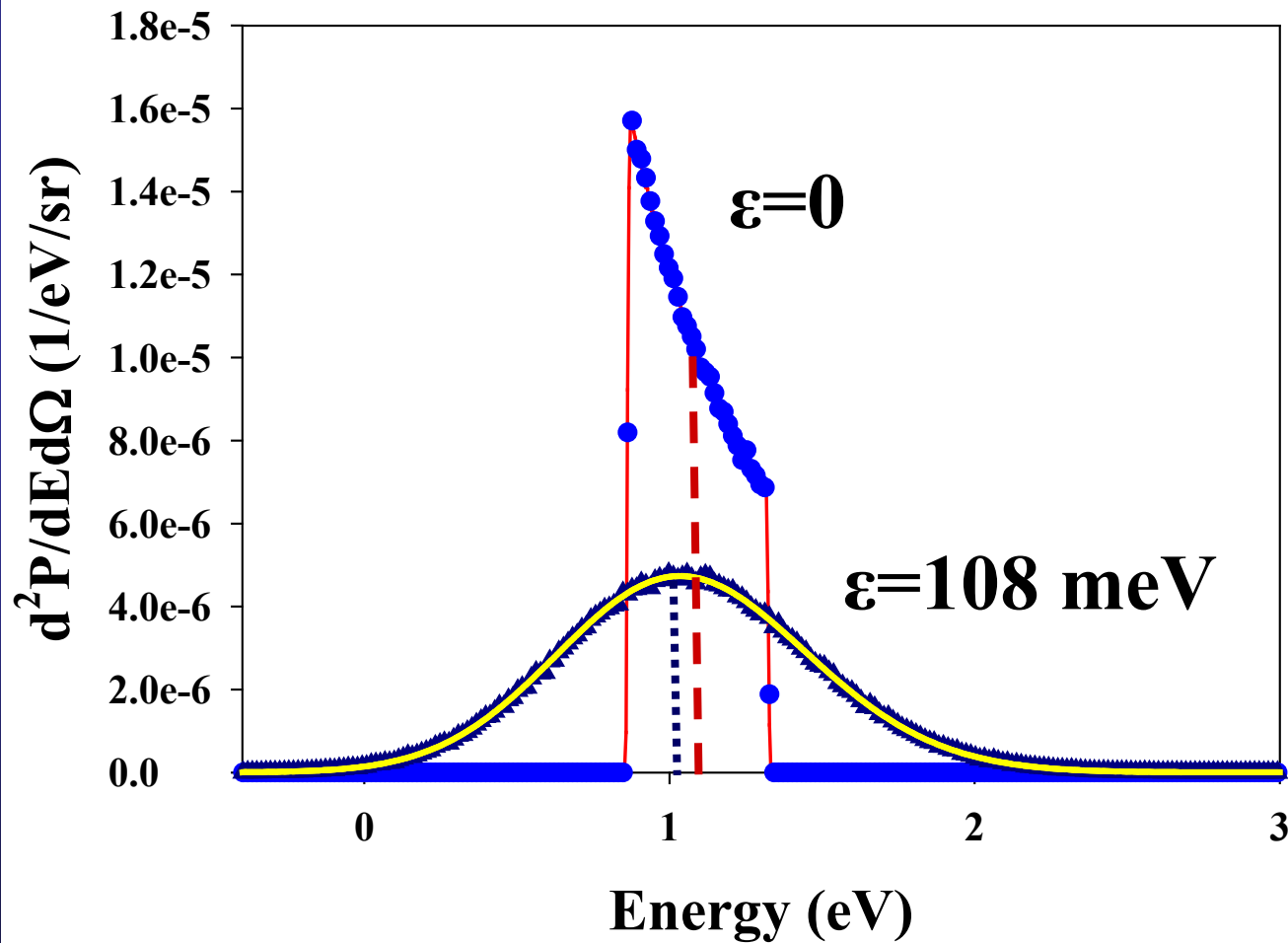


Solid line : Au – 1 Å, C – 90 Å

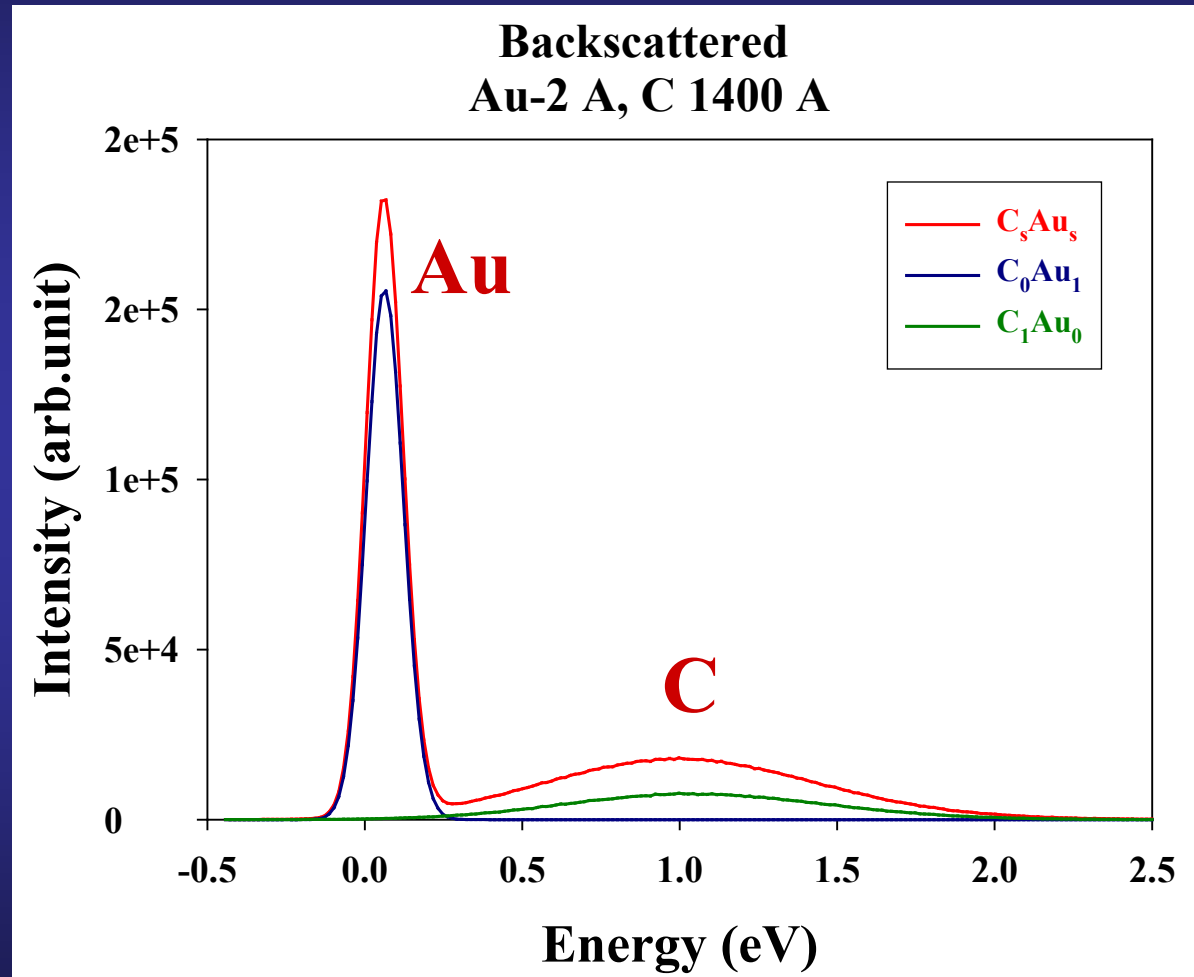
Dashed line: Au – 2 Å, C – 1400 Å

# Single scattering on carbon

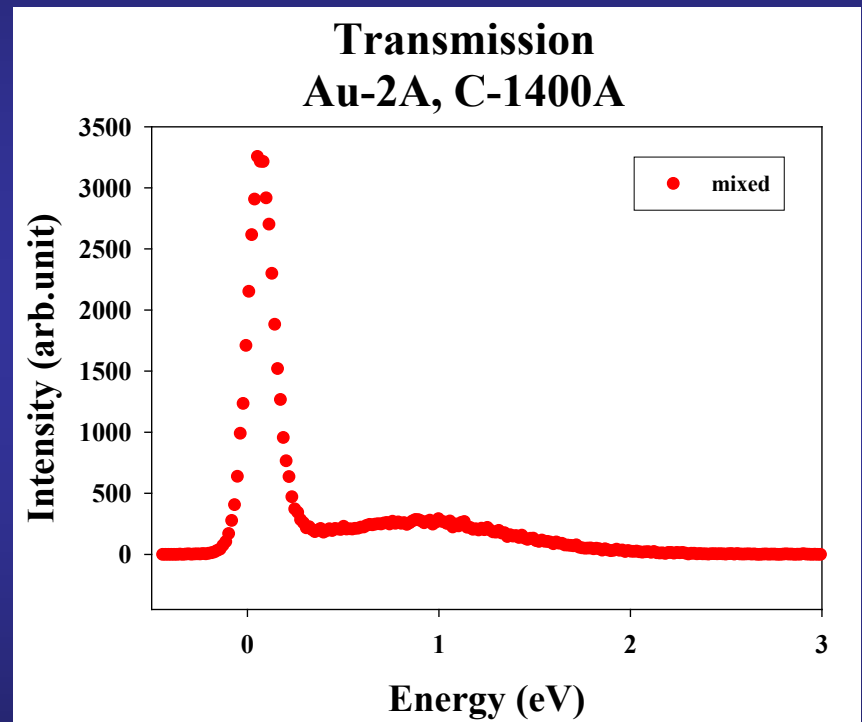
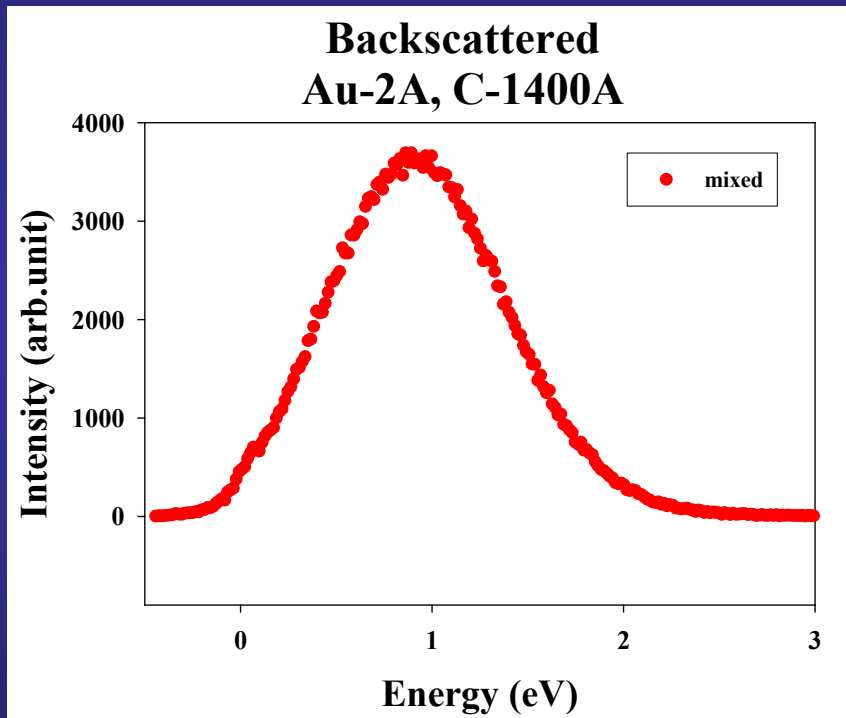
1-90 Au-C 40 keV - C  
Backscattered



# Partial energy loss distributions at 40 keV primary energy $\theta_0 = 44.3^\circ$ and $\Delta\Omega = \pm 5^\circ$ solid angle



# Multiple scattering on different components ( $C^iH^j$ $i \geq 1, j \geq 1$ ) at 40 keV primary energy



# Partial Elastic scattering yields - I

Geometry [sample] (Å)	Peaks —— Sum	Single scattering (%)	Multiple scattering	
			Same atomic mass (%)	Different atomic mass (%)
Reflection [Au-1] [C-90]	Au	49.35	3.98	(≤ 0.1)
	C	29.07	11.84	(~ 5.7)
	Σ	<b>78.42</b>	<b>15.82</b>	<b>5.76</b>
Reflection [Au-2] [C-1400]	Au	50.10	8.03	(≤ 0.1)
	C	16.87	15.88	(~ 9.1)
	Σ	<b>66.96</b>	<b>23.91</b>	<b>9.13</b>
Transmission [Au-1] [C-90]	Au	35.86	0.83	(6.11)
	C	47.32	8.33	(1.54)
	Σ	<b>83.19</b>	<b>9.16</b>	<b>7.65</b>
Transmission [Au-2] [C-1400]	Au	0.68	0.03	(7.32)
	C	7.11	79.98	(4.87)
	Σ	<b>7.79</b>	<b>80.02</b>	<b>12.19</b>

# Elastic peak intensities

Geometry [sample] (Å)	P <sub>1</sub> (C) / P <sub>1</sub> (Au)			P <sub>mono</sub> (C)/P <sub>mono</sub> (Au) Monte Carlo	P(C)/P(Au) Monte Carlo	P(C)/P(Au) ————— P <sub>1</sub> (C)/P <sub>1</sub> (Au)
	Θ <sub>0</sub> =44.3° (dΩ)	Integrated (ΔΩ)	Monte Carlo (ΔΩ)			
<b>Reflection</b> [Au - 1] [C - 90]	0.5939	0.5895	0.589 (1)	0.767 (2)	0.875 (3) 0.883 (16)	1.48 1.50
<b>Reflection</b> [Au - 2] [C - 1400]	0.3417	0.3365	0.337 (1)	0.563 (1)	0.720 (2) 0.730 (8)	2.14 2.17
<b>Transmission</b> [Au - 1] [C - 90]	1.3071	1.3070	1.320 (5)	1.517 (5)	1.336 (6) 1.343 (10)	1.01 1.02
<b>Transmission</b> [Au - 2] [C - 1400]	10.167	10.167	10.43 (24)	122 (2)	11.45 (14) 12.75 (41)	1.12 1.25

*Test of the Monte Carlo simulation*

# Elastic peak positions

Single scattering:

$$E_0 = 40\text{keV} \quad E_{r0}(\text{C}) = 1080\text{meV}$$

$$\Theta_0 = 44.3^\circ \quad E_{r0}(\text{Au}) = 66\text{meV}$$

---


$$E_{r0}(\text{C}) - E_{r0}(\text{Au}) = 1014\text{meV}$$

Geometry [sample] (Å)	$E_{r0}(\text{C}) - E_{r0}(\text{Au})$ [meV]		
	Single scattering	Mono atomic scattering	Sum of the scatterings
<b>Reflection</b> [Au - 1] [C - 90]	984 (1)	968 (1)	952 (1) (-6.1%)
<b>Reflection</b> [Au - 2] [C - 1400]	988 (1)	965 (1)	940 (1) (-7.3%)
<b>Transmission</b> [Au - 1] [C - 90]	977 (1)	974 (1)	970 (1) (-4.3%)
<b>Transmission</b> [Au - 2] [C - 1400]	968 (3)	926 (2)	910 (2) (-10.3%)



# Elastic peak widths

Single scattering:

$$E \quad E_0 = 40 \text{ keV}$$

$$\Theta \quad \Theta_0 = 44.3^\circ \quad \rightarrow \Delta E_r(\text{Au}) = 149 \text{ meV}$$

$$- \quad \bar{\varepsilon}(\text{Au}) = 40 \text{ meV}$$

Geometry [sample] (Å)	$\Delta E_r$ (C) [FWHM ; meV]		
	Single scattering	Mono atomic scattering	Sum of the scatterings
<b>Reflection</b> [Au - 1] [C - 90]	966 (2)	979 (2)	1006 (2) (+4.1%)
<b>Reflection</b> [Au - 2] [C - 1400]	970 (2)	996 (2)	1034 (2) (+6.6%)
<b>Transmission</b> [Au - 1] [C - 90]	959 (2)	963 (2)	985 (3) (+2.7%)
<b>Transmission</b> [Au - 2] [C - 1400]	954 (5)	1020 (4)	1061 (4) (+11.2%)

**Quantitative analysis of the hydrogen  
peak in the spectra of electrons  
backscattered from polyethylene**

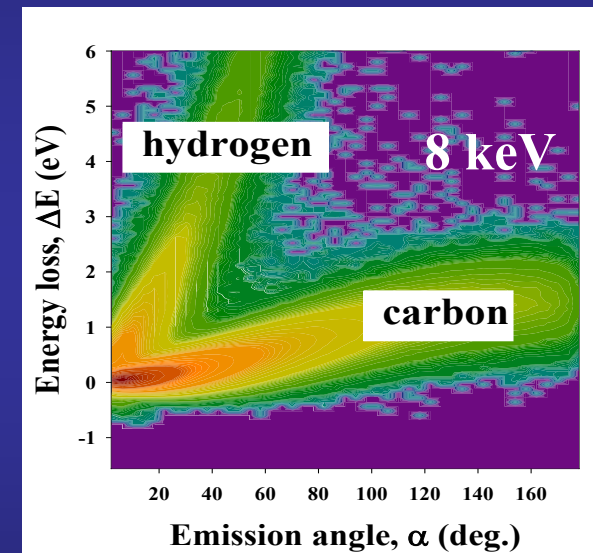
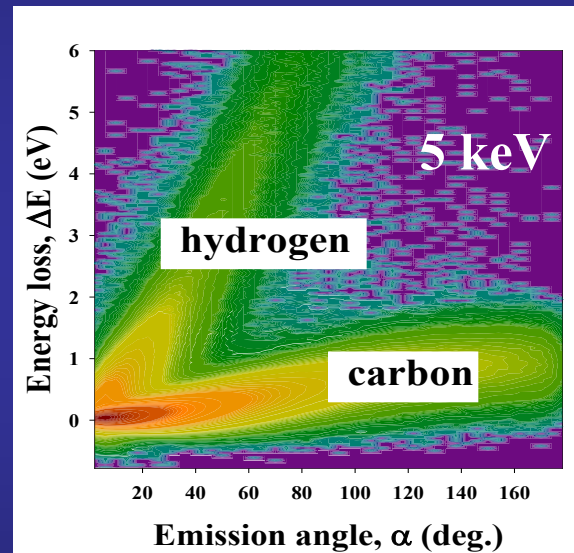
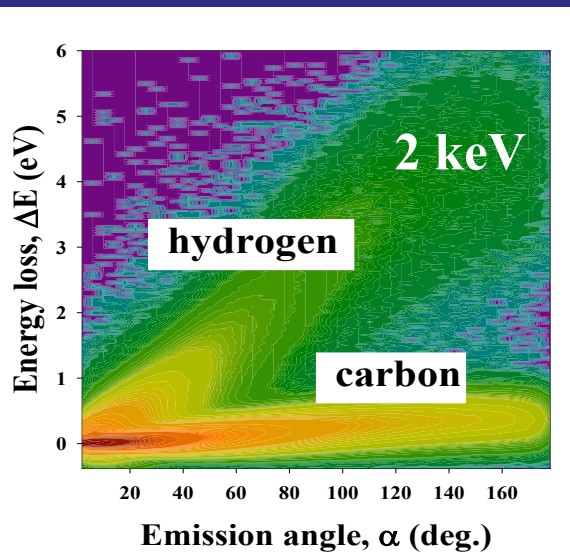
## Background

- Observation of the hydrogen peak is a challenging if not impossible task in conventional electron spectroscopy.
- Hydrogen was observed earlier in formvar film in high energy electron scattering experiments using transmission geometry.

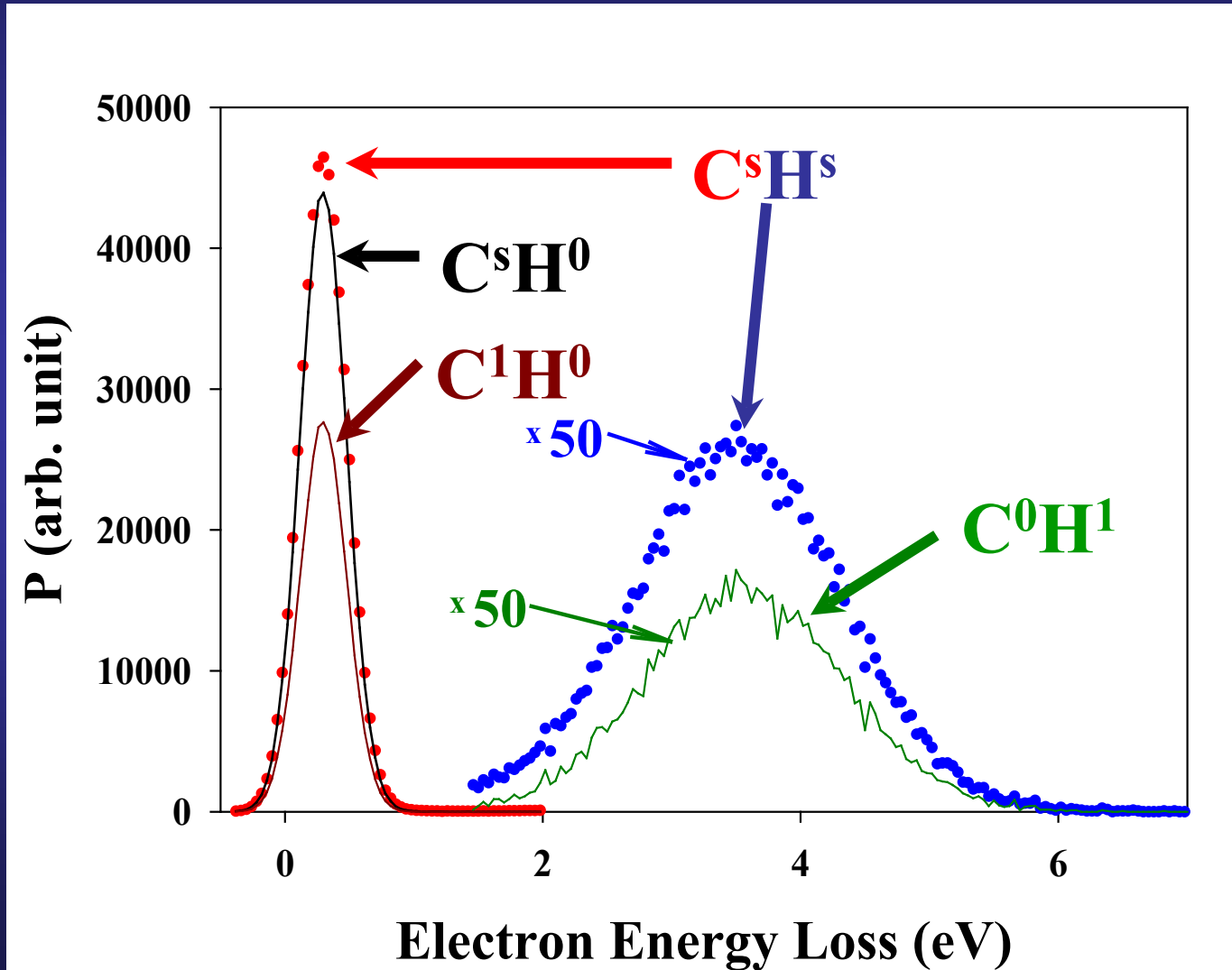
## Motivation

- An alternative way for the detection of hydrogen is shown.
- The spectra of electrons backscattered elastically from polyethylene  $(\text{CH}_2)_n$  is used.

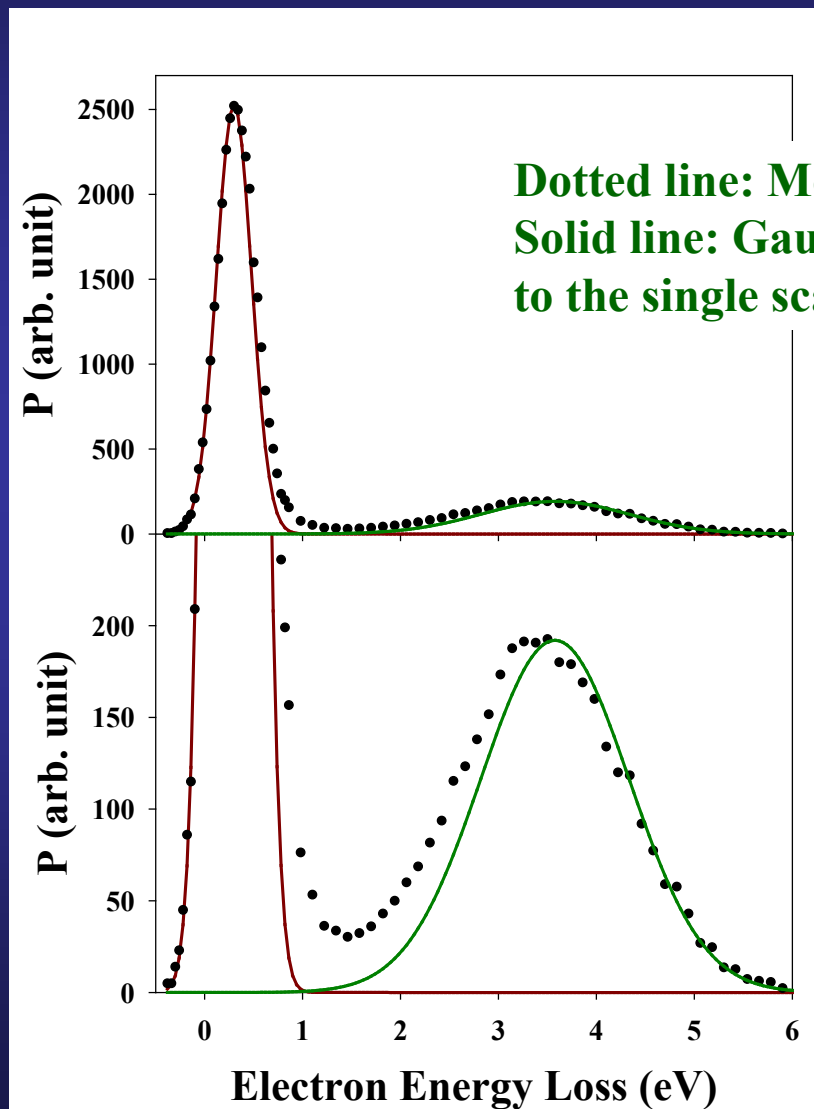
Contour plot (blue: minimum intensity, red: maximum intensity) of the electron intensity backscattered elastically from polyethylene.



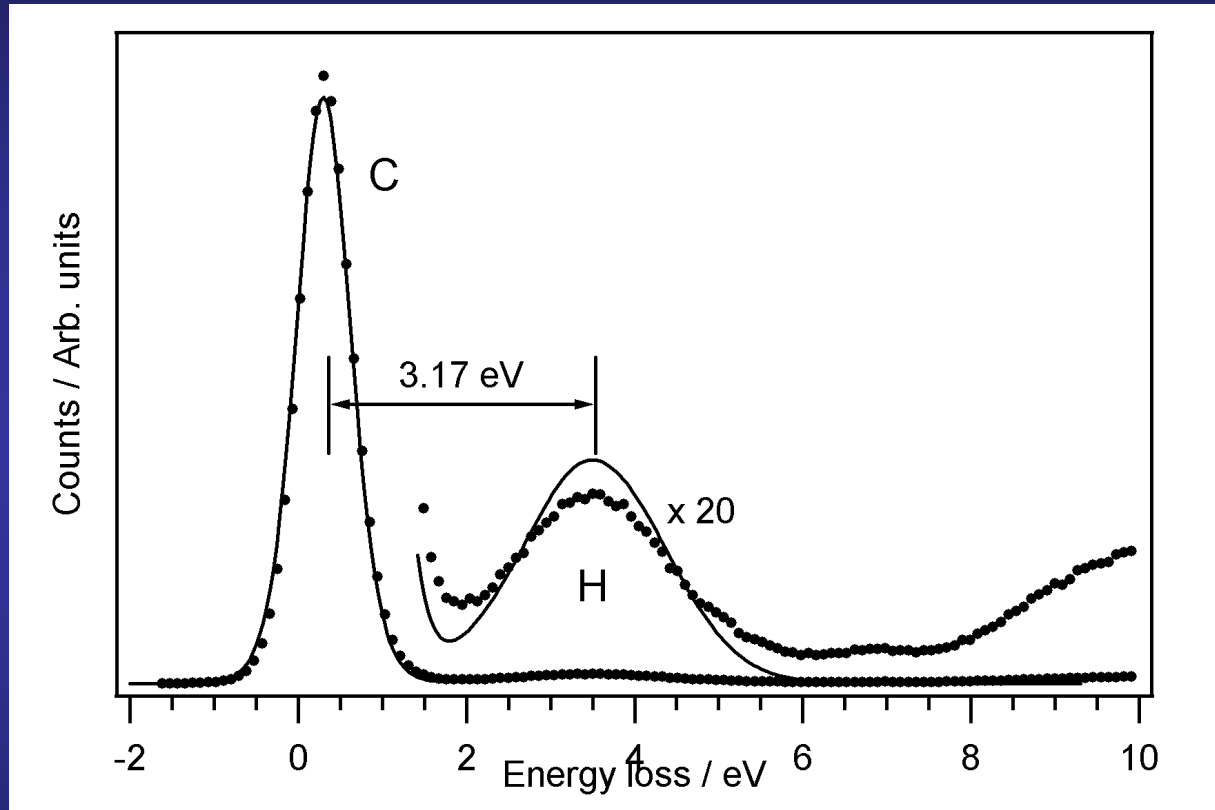
# Energy loss distributions at 2 keV primary energy $\theta = 130^\circ$ and $\Delta\Omega = \pm 5^\circ$ solid angle



# Multiple scattering on different components ( $C^iH^j$ $i \geq 1, j \geq 1$ ) at 2 keV primary energy



# Comparison with experiment



The energy distribution of electrons backscattered elastically from polyethylene at  $\theta = 130^\circ$ . The primary electron energy was 2 keV. Dotted line: measurement, solid line: MC simulation.

# CONCLUSIONS

- The high energy resolution elastic peak electron spectroscopy is applicable for quantitative analysis.
- The relative hydrogen content can be estimated by the observation of the well separated *H* elastic peak.
- The single scattering model can be used for thin samples and transmission geometry.
- However, the multiple scatterings, depending on the primary electron energy and geometry, modify the measurable data (distance between the elastic peaks, FWHM, yields), therefore for more precise analysis the contribution of mixed multiple electron scattering has to be taken into account – **Monte Carlo simulation.**



**THANK YOU FOR  
YOUR ATTENTION!**